SECTION F2

LAMINATED COMPOSITES

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F2.0 STRENGTH OF LAMINATED COMPOSITES.

The strength of laminated composites must be related to the individual lamina [1]. This is because it is easier to determine where nonlinearity and degradation begin to occur on a unidirectional test specimen than on some general laminate test. For this reason the trend in determining the strength of advanced composites is to establish strength allowables for the orthotropic lamina and then to utilize analytical methods to predict the yield or the ultimate strength of the laminate.

2.1 YIELD STRENGTH.

The available yield strength theories for laminated composites are at best tentative at this time [2]. Only a minimal amount of test data is available to substantiate any of the yield theories for advanced composites.

If the yield point is defined as the onset of inelastic action, it is apparent that the prediction of the yield strength of an orthotropic lamina is a linear problem. Several yield theories of failure have been hypothesized for anisotropic materials, of which two will be discussed (the distortional energy theory and the maximum strain theory).

Before the discussion of yield theories, the basic difference between the yield surfaces for an isotropic and for an orthotropic or anisotropic material must be explained. For an isotropic material, any biaxial stress state, σ_x, σ_y, τ_{xy}, may be resolved into two principal stresses, σ_{1_p} and σ_{2_p}, and some angle, θ; therefore, a plot of the principal stresses that cause yield will give the required yield surface. The result is a two-dimensional figure with σ_{1_p} and σ_{2_p} as axes. When considering the yield surface for an orthotropic lamina, the stresses must be referred to the lamina principal axes; therefore, for biaxial stress states three stress components may appear in the yield criteria. The resulting yield surface will appear as a three-dimensional figure with the directions of σ_1, σ_2, and τ_{12} as reference coordinates (Fig. F2.1-1).

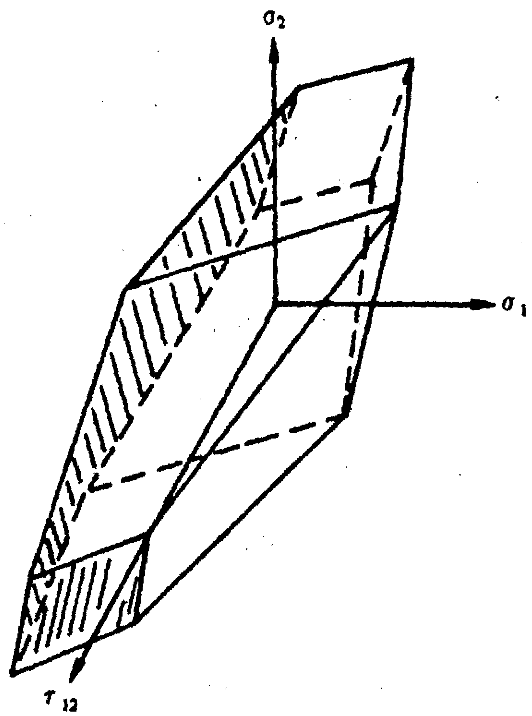

FIGURE F2.1-1. THREE-DIMENSIONAL YIELD SURFACE FOR AN ORTHOTROPIC LAMINA

2.1.1 DISTORTIONAL ENERGY THEORY.

Cira, Norris, and Hill independently developed their generalizations of the von Mises isotropic distortional energy yield criterion,

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + 6\,(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2) = 2\sigma_0^2 \,, \quad \text{(F2.1-1)}$$

to account for the anisotropy of their respective problems [3]. Hill was concerned with the tendency of isotropic metals to exhibit certain anisotropic properties when undergoing metal working involving severe strains. Hill claimed he had a physical interpretation of von Mises' "plastic potential," which allowed him to generalize von Mises' yield criterion for application to anisotropic metals. The plastic potential or yield criterion has the form

$$2f\,(\sigma_{ij}) = A_1\,(\sigma_2 - \sigma_3)^2 + A_2\,(\sigma_3 - \sigma_1)^2 + A_3\,(\sigma_1 - \sigma_2)^2 + 2A_4\tau_{23}^2 + 2A_5\tau_{31}^2 + 2A_6\tau_{12}^2 = 1 \quad \text{(F2.1-2)}$$

where

$$2f\,(\sigma_{ij}) = \text{the plastic potential},$$

$$2A_1 = (F_2)^{-2} + (F_3)^{-2} - (F_1)^{-2} \;,$$

$$2A_2 = (F_3)^{-2} + (F_1)^{-2} - (F_2)^{-2} \;,$$

$$2A_3 = (F_1)^{-2} + (F_2)^{-2} - (F_3)^{-2} \;,$$

$$2A_4 = (F_{23})^{-2} \;,$$

$$2A_5 = (F_{31})^{-2} \;,$$

and

$$2A_6 = (F_{12})^{-2} \;,$$

F_1, F_2, and F_3 are determined from either uniaxial tension or compression tests, and F_{12}, F_{23}, and F_{31} are determined from pure shear tests.

Tsai [2] adapted this criterion as a yield and failure criterion for laminated composites. The failure criterion for a laminated composite is based on the strengths of the individual orthotropic lamina referred to the lamina principal axes since the yield strengths are established experimentally with reference to these axes. The Hill criterion reduces to

$$\left(\frac{\sigma_1}{\sigma_{1_y}}\right)^2 - \frac{1}{r}\,\frac{\sigma_1}{\sigma_{1_y}}\,\frac{\sigma_2}{\sigma_{2_y}} + \left(\frac{\sigma_2}{\sigma_{2_y}}\right)^2 + \left(\frac{\tau_{12}}{\tau_{12_y}}\right)^2 = 1 \qquad \text{(F2. 1-3)}$$

for an orthotropic material in plane stress. Note that

$$\frac{1}{r} = \frac{\sigma_{1_y}}{\sigma_{2_y}} \;. \qquad \text{(F2. 1-4)}$$

Also, σ_{1_y} and σ_{2_y} are the tensile or compressive yield strengths in the 1 and 2 directions for the orthotropic lamina, and τ_{12_y} is the shear yield stress. The yield surface is either an ellipsoid or sphere, depending on r, when plotted in a three-dimensional space (Fig. F2. 1-2).

2. 1. 2 MAXIMUM STRAIN CRITERIA.

The maximum strain yield criteria presented here should not be confused with the maximum principal strain yield criteria for isotropic materials [2]. The strain components in the orthotropic lamina must be referred to the lamina principal axes; therefore, it is possible for three strain components to appear in the yield criteria.

The maximum strain yield criteria may be developed from equation (F1. 2-3) with the strains equal to the yield strains:

$$\begin{bmatrix} \epsilon_{1_y} \\ \epsilon_{2_y} \\ \gamma_{12_y} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} . \qquad (F2.\ 1\text{-}5)$$

Thus, equation (F2. 1-5) gives the envelope of stresses which produce the yield strains in the lamina. A two-dimensional plot of the equation may be made if $\tau_{12} = 0$ is assumed. Then,

$$\epsilon_{1_y} = S_{11}\sigma_1 + S_{12}\sigma_2$$

and (F2. 1-6)

$$\epsilon_{2_y} = S_{12}\sigma_1 + S_{22}\sigma_2 \quad ,$$

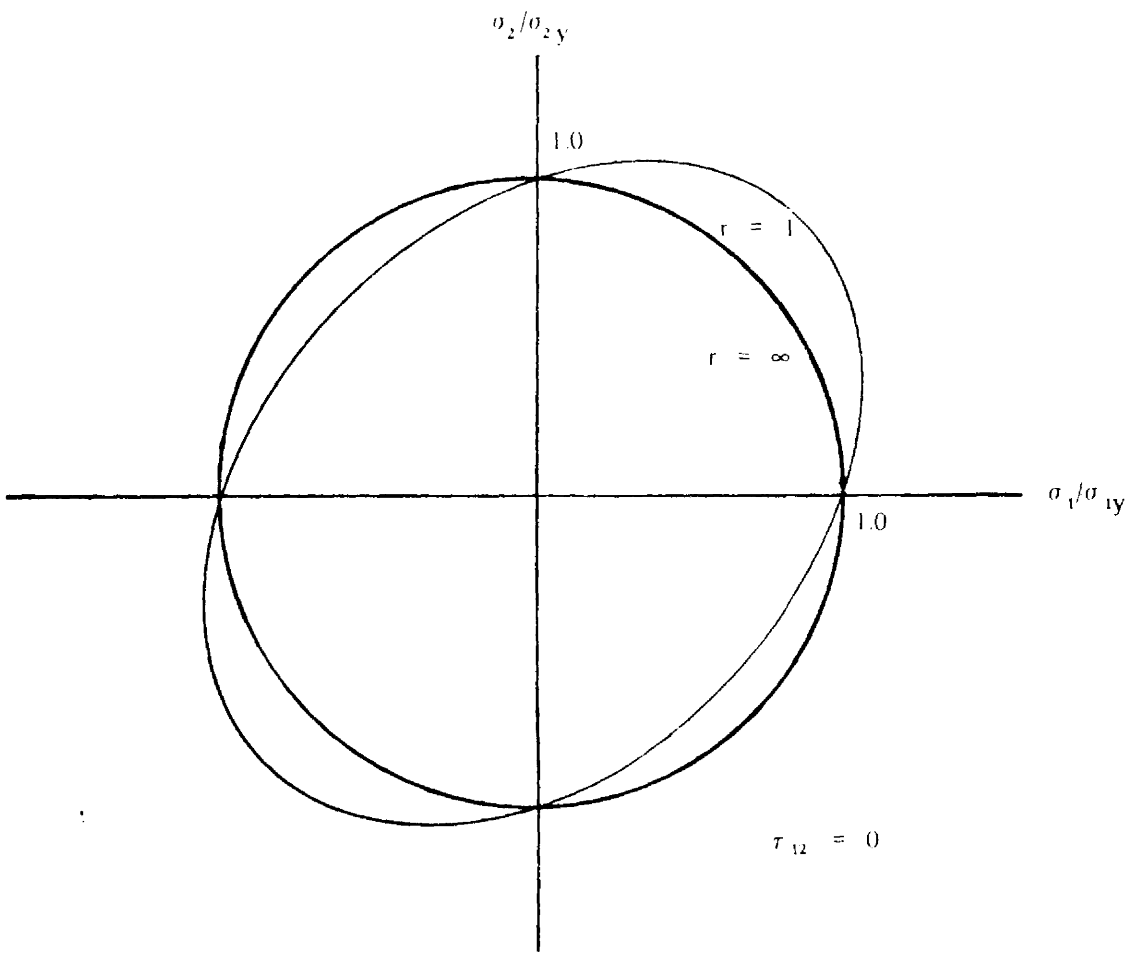

FIGURE F2.1-2. HILL YIELD SURFACE

or

$$\sigma_2 = \frac{\epsilon_{1_y}}{S_{12}} - \frac{S_{11}}{S_{12}} \sigma_1$$

and (F2. 1-7)

$$\sigma_2 = \frac{\epsilon_{2_y}}{S_{22}} - \frac{S_{12}}{S_{22}} \sigma_2 \quad .$$

Equations (F2. 1-7) are the equations of two lines in the $\sigma_1 - \sigma_2$ coordinate system which define yield of an orthotropic lamina. After defining the yield strengths as

$$\sigma_{1_y} = \frac{\epsilon_{1_y}}{S_{11}} = \epsilon_{1_y} E_{11}$$

and (F2. 1-8)

$$\sigma_{2_y} = \frac{\epsilon_{2_y}}{S_{22}} = \epsilon_{2_y} E_{22} \quad ,$$

equations (F2. 1-7) may be put in a form similar to the Hill equation:

$$\left(\frac{\sigma_2}{\sigma_{2_y}}\right) = \frac{\epsilon_{1_y}}{\epsilon_{2_y}} \frac{S_{22}}{S_{12}} - \frac{\epsilon_{1_y}}{\epsilon_{2_y}} \frac{S_{22}}{S_{12}} \left(\frac{\sigma_1}{\sigma_{1_y}}\right)$$

and (F2. 1-9)

$$\left(\frac{\sigma_2}{\sigma_{2_y}}\right) = \frac{\epsilon_{2_y}}{\epsilon_{2_y}} - \frac{\epsilon_{1_y}}{\epsilon_{2_y}} \frac{S_{12}}{S_{11}} \left(\frac{\sigma_1}{\sigma_{1_y}}\right) \quad .$$

Equations (F2.1-7) may be plotted on a $\sigma_1 - \sigma_2$ set of coordinates since it was assumed that the shear stress component, τ_{12}, was zero. Such a plot might appear as shown in Fig. F2.1-3.

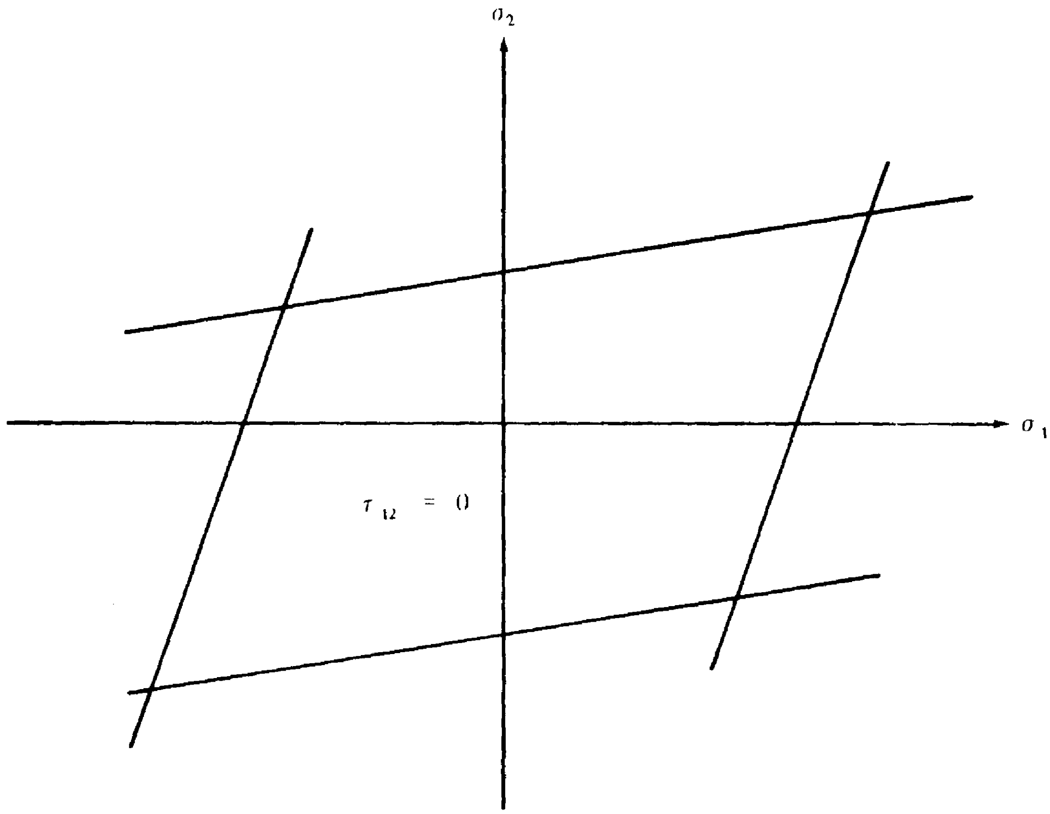

FIGURE F2.1-3. LAMINA YIELD SURFACE

2.2 ULTIMATE STRENGTH.

Until recently, attempts to predict the ultimate or rupture load for a laminate were based on linear theory; that is, the laminate ultimate load was based upon the assumption that the laminate stress-strain response is linear to failure or that upon yield of a constituent lamina, some (or all) of the lamina moduli are reduced to some arbitrarily small value or set equal to zero. Recently, some computer programs have been written with relatively simple techniques, and new methods that determine the ultimate strength of laminates have been developed. These programs and methods are usually for specialized laminates, and sufficient test data are not available at this time to verify their accuracy for general use.

2.3 REFERENCES.

1. Ashton, J. C; Halpin, J. C.; and Petit, P. H.: Primer on Composite Materials: Analysis. Technomic Publishing Co., Inc., Stamford, Conn., 1969.

2. Petit, P. H.: Basic Concepts in the Design and Stress Analysis of Laminated Composites. Report No. SMN 239, Lockheed-Georgia Co., a division of Lockheed Aircraft Corp., Marietta, Ga., March 1968.

3. Kaminski, B. E.; and Lantz, R. B.: Strength Theories of Failure for Anisotropic Materials. Composite Materials: Testing and Design. Report STP 460, American Society of Testing Materials, 1969.