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16. ABSTRACT

This document (Volumes I, II, and III) presents a compilation of industry-wide methods in aerospace strength analysis that can be carried out by hand, that are general enough in scope to cover most structures encountered, and that are sophisticated enough to give accurate estimates of the actual strength expected. It provides analysis techniques for the elastic and inelastic stress ranges. It serves not only as a catalog of methods not usually available, but also as a reference source for the background of the methods themselves.

An overview of the manual is as follows: Section A is a general introduction of methods used and includes sections on loads, combined stresses, and interaction curves; Section B is devoted to methods of strength analysis; Section C is devoted to the topic of structural stability; Section D is on thermal stresses; Section E is on fatigue and fracture mechanics; Section F is on composites; Section G is on rotating machinery; and Section H is on statistics.

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SECTION D

THERMAL STRESSES

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DEFINITION OF SYMBOLS

Symbol	Definition
A	Cross-sectional area; area
A_0	Cross-sectional area of beam at $x = 0$
A_{mn}, A_{pq}	Coefficients for the series by which the stresses are expressed, in.
A_1, A_2, A_3, A_4	Constants based on the boundary conditions, equations (95) and (96), dimensionless
A_i', A_1', A_2', A_3'	Constants, psi (Figs. 5.0-8, 5.0-9)
a	Limiting value (lower) for radius; inside radius or radius of middle surface of cylinder
a_0	Maximum value of initial imperfection
a_0'	Constant, °F
a_1'	Constant, °F/in.
B_{mn}, B_{pq}	Coefficients for the series by which the stresses are expressed, in.

NOTES:

1. Bars over any letters denote middle-surface values.
2. The subscript cr denotes critical values for buckling.
3. The superscripts P and C identify quantities associated with the particular and complementary solutions, respectively.
4. The subscript R denotes values required to completely suppress thermal deformations.

Symbol	Definition
B_1, B_2, B_3	Constants, in./(in.)(hr)(Fig. 5.0-10)
b	Breadth (or width) of cross section; limiting value (upper) for radius; outside radius
b_0, b_1, b_2	Constants in polynomial representation of the temperature $T_1(x)$; °F, °F/in., and °F/in.^{2}, respectively
C_p	Specific heat of the material, Btu/(lb)(°F)
C_1, C_2, C_3, C_4	Constants of integration, in.
$C_{-1}, C_0, C_1, \ldots$	Coefficients in polynomial representation of U^P, in.-lb, lb, lb/in., . . . , respectively, refer to equation (106)
D	Diameter
D_b	Plate bending stiffness or shell-wall bending stiffness
d_0, d_1, d_2	Constants in polynomial representation of the function $T_2(x)$; °F, °F/in., and °F/in.^{2}, respectively
$d_{-1}, d_0, d_1, \ldots$	Coefficients in polynomial representation of V^P, in., dimensionless, 1/in., . . . , respectively; refer to equation (106)
E	Young's modulus of elasticity
E_b	Young's modulus of support-beams, psi
E_p	Young's modulus of plate, psi

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
E_s	Secant modulus, psi
E_t	Tangent modulus, psi
e	Base for natural logarithms, dimensionless (2.718)
F.E.M.	Fixed-end moment
FF	Fixed-fixed
FS	Fixed-supported
G	Variation in depth of beam along the length
G'	Modulus of rigidity or shear modulus
H	Variation in width of beam along the length
H_A, H_B	Running edge forces acting normal to the axis of revolution at positions A and B, respectively (Figs. 3.0-51 and 3.0-52), lb/in.
I	Moment of inertia
I_b	Support-beam centroidal moment of inertia
I_y, I_z	Area moments of inertia taken about the y and z axes, respectively, $in.^4$
i	Imaginary number, $\sqrt{-1}$
K	Thermal diffusivity of the material, $ft^2/hr = k/C_p \rho$
k	An integer (1, 2, 3, 4, 5) exponent

Symbol	Definition
k'	Thermal conductivity of the material, Btu/(hr)(ft)(°F)
L	Length
$L(\ )$	Operator defined by equation (103)
M	Moment
M_A, M_B	Running edge moments acting at positions A and B, respectively (Figs. 3.0-51 and 3.0-52), in.-lb/in.
M_T	Thermal moment
M_b	Thermal bending parameter, in.-lb/in.
$M_r, M_\theta, M_x, M_\phi$	Running bending moments, in.-lb/in.
$M_{r\theta}$	Running twisting moment, in.-lb/in.
M_r', M_θ'	Bending-moment parameters (Table 3.0-5 and Figs. 3.0-15 through 3.0-19)
M_t	Temperature resultant, in.-lb/in.
M_x, M_y	Running bending moments acting on sections of the plate which are perpendicular to the x and y directions, respectively (positive when associated upper-fiber stresses are compressive), in.-lb/in.
M_y	Moment about y axis
M_z	Moment about z axis
M_0	Moment in beam at $x = 0$

Symbol	Definition
m	Temperature distribution in the z-direction
m_k	Moment coefficients, plotted in Figure 3.0-46, dimensionless
m_ϕ	Surface moment (Fig. 3.0-53) in.-lb/in.^{2}
N	Exponent of thermal variation along the length of the beam; also upper limit for summation indices, dimensionless
N_T	Axial load per unit length on plate edge
$N_r, N_\theta, N_x, N_\phi$	Running membrane loads, lb/in.
$N_{r\theta}$	Running membrane shear load, lb/in.
N_r', N_θ'	Membrane-force parameters (Table 3.0-6), dimensionless
N_t	Temperature resultant, lb/in.
n	Temperature distribution in the y-direction
n_k	Hoop-force coefficients, plotted in Figure 3.0-49, dimensionless
P	Axial force
P_T	Axial force resulting from temperature
P_0	Column load
p	Radial pressure, psi
p, q	Summation indices, dimensionless
Q	Heat input

Symbol	Definition
Q_x	Running transverse shear load, lb/in.
q	Temperature distribution in the x-direction
q_k	Shear coefficients, plotted in Figure 3.0-46, dimensionless
r	Radius
SS	Simply supported
s	Meridional coordinate measured downward from top of the truncated cone (Fig. 3.0-50), in.
s^*	Meridional coordinate measured upward from bottom of the truncated cone (Fig. 3.0-50), in.
T	Temperature
$\bar{T}$	Average value for T, °F
T^*	Weighted average value for T, °F
T_D	Temperature difference between the plate faces, °F
T_{edges}	Temperature at edges of the plate, °F
T_f	Final uniform temperature which the body reaches at sufficiently long times
T_i	Inside temperature; also initial uniform temperature of the body, °F
T_m	Average value for temperature distribution across the wall thickness at any single position, °F

Symbol	Definition
T_s	Temperature of the supports, °F
T_{xy}	Temperature at any location in the plate, °F
T_0	Outside temperature
T_1, T_2	Temperature functions, °F
t	Time (hr) or thickness
t_{cr}	Time to the onset of creep buckling, hr
u	Displacement in the x-direction or r-direction for circular plate
V	Function representing temperature variation in y- and z-directions; also rotations in a meridional plane for a shell
V_P	Component of deflection without thermal effects
V_T	Component of deflection including thermal effects
V_0	Shear at $x = 0$
v	Displacement in the y-direction or θ-direction for circular plate
w	Displacement in the z-direction
w'	Deflection parameter (Table 3.0-5 and Figs. 3.0-15 through 3.0-19), dimensionless

Symbol	Definition
$\overline{w}$	Displacement, in the z-direction, for the case where all edges are simply supported, in.; also radial deflection for shell
w^A	Displacement component, in the z-direction, in. (Note: The superscript A is merely an identification symbol and is not meant to be a generalized exponent.)
w_k	Deflection coefficients, plotted in Figure 3.0-45, dimensionless
x	Coordinate axis
y	Coordinate axis
Z	Upper limit for the summation index k, dimensionless; also surface loads, psi
z	Coordinate axis measured normal to undeformed plate
α	Coefficient of linear thermal expansion, in./(in.)(° F)
$(\alpha T_0 b^2/t^2)_{cr}$	Critical value of temperature parameter (value at which initial thermal buckling occurs), dimensionless
Γ	Knockdown factor, dimensionless
γ	Knockdown factor (Fig. 4.0-17), dimensionless
γ_{xy}	Shearing strain in planes parallel to and including the x-y plane, in./in.
$\dot{\gamma}_{xy}$	Time rate of change for γ_{xy}, in./(in.)(hr)

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
δ	Maximum absolute value for deflection measured normal to the x-y plane, in.
∇	Del-operator
ϵ	Unit strain
ϵ_i	Strain intensity defined in equations (1), in./in.
$\dot{\epsilon}_i$	Time rate of change for ϵ_i, in./(in.)(hr)
ϵ_x, ϵ_y	Normal strains acting in the x and y directions, respectively (positive when fibers lengthen), in./in.
$\dot{\epsilon}_x, \dot{\epsilon}_y$	Time rate of change for ϵ_x and ϵ_y, respectively, in./(in.)(hr)
$\zeta(\;)$	Function defined by equations (56) and (76), dimensionless
η	Plasticity reduction factor, dimensionless
θ	Angular coordinate (Fig. 3.0-14), rad
$\theta(\;)$	Function defined by equations (58) and (78), dimensionless
θ_k	Slope coefficients, plotted in Figure 3.0-46, dimensionless
λ	A constant in strain-stress relationship
ν	Poisson's ratio (sometimes written μ, m)
ν_1	$\nu/(1-\nu)$
ρ	Density of the material, lb/ft^3

Symbol	Definition
σ_f	Stress induced by restraint
σ_i	Stress intensity defined in equations (1), psi
$(\sigma_i)_{cr}$	Critical value for the stress intensity σ_i, psi
$(\sigma_{\overline{P}})_B$	Axial stress due to the artificial force $\overline{P}_B$, psi
$\sigma_r, \sigma_t, \sigma_\theta, \sigma_\phi$	Normal stresses acting in the r, t, θ, and ϕ directions, respectively (positive in tension), psi
$\sigma_{r\theta}$	In-plane shear stress, psi
σ_x, σ_y	Normal stresses acting in the x and y directions, respectively (positive in tension), psi
$(\sigma_x)_{cr}$	Critical axial stress for buckling of the cylinder, psi
σ_{yy}, σ_{zz}	Lateral axial stresses
σ_{yz}	Plane stress
τ_{xy}	Shearing stress acting in planes parallel to and including the x-y plane, psi
ϕ	Stress function [Airy's stress function I(x,y)]; also denotes "meridional"; also angular coordinate
$\phi(\ )$	Function defined in equations (76), dimensionless
ϕ_1, ϕ_2, ϕ_3	Parameters tabulated in Tables 6.0-1, 6.0-2, and 6.0-4, respectively, dimensionless

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
Ψ_2, Ψ_3	Parameters tabulated in Tables 6.0-3 and 6.0-5, respectively, dimensionless
Ψ_1'	Parameter tabulated in Table 6.0-1, dimensionless
Ψ_2'	Value of Ψ_2 at $r/R = 1$, dimensionless
Ψ_3'	Value of Ψ_3 at $r/R = 1$, dimensionless
$\psi(\ )$	Function defined in equations (78), dimensionless

D. THERMAL STRESSES.

1.0 INTRODUCTION.

Restrictions imposed on thermal expansion or contraction by continuity of the body or by the conditions at the boundaries induce thermal stresses in the body. In the absence of constraints at boundaries, thermal stresses in a body are self equilibrating.

Except for a few simple cases, the solution of the thermoelasticity problem becomes intractable (see Ref. 1). Therefore, for thermal stress analysis, further approximations leading to the strength of material and finite element methods are used extensively. Depending upon its geometry, a structural element is classified as one of the following: rod, beam, curved beam, plate, or shell. If a structure consists of one of the elements named above, or of some simple combination of them, the method of strength of materials will yield good results. However, if the structure has a complex geometrical shape, the finite element method is easier to use and yields satisfactory results. The method of finite element analysis is suggested for use on an idealized structure which can be represented by a large number of smaller, simpler elements (rods, beams, triangular plates, rectangular plates, etc.) connected at a finite number of points (e.g., only at vertices of triangles or rectangles, or ends of rods, etc.) to provide approximately the configuration of the actual structure.

In a constrained structure, compressive stresses resulting from thermal, or thermal and mechanical, loading may produce instability of the structure. The linear thermoelastic formulation of the problem excludes the question of large deformations. Thus, for buckling, or for problems where loads depend upon deformation, nonlinearity that is due to large deformations must be incorporated in the problem formulation (e.g., beam-column analysis). The extreme difficulties involved in solving the nonlinear thermoelasticity problem have led the researchers to resort to the approximate methods of strength of materials and finite elements.

One of the important problems associated with high temperature is that of creep deformation and relaxation. The phenomenon of the increase in strains with time when the specimen is subject to constant stress and constant high temperature is called creep. The general formulation remains the same as in thermoelasticity or strength of materials, except that the stress-strain relation is expressed by a viscoelastic model. The linear viscoelastic model does not represent many materials; but the complexities multiply if the nonlinear model is used. Relatively little work has been done towards the solution of nonlinear viscoelastic theory.

Vibrations that result from thermal shock are quite small in comparison with those resulting from mechanical load. They are not considered here.

2.0 THERMOELASTICITY.

Three-dimensional equations for equilibrium, displacement, stress, and strain can be found in Ref. 1 in terms of rectangular, cylindrical, or spherical coordinates. Formulas given below are, for the most part, two-dimensional expressions for rectangular coordinates.

2.0.1 Plane Stress Formulation.

For a temperature distribution of the form $\dot{T}(x, y)$ in a long prismatic body, eight quantities, σ_{xx}, σ_{yy}, σ_{xy}, ϵ_{xx}, ϵ_{yy}, ϵ_{xy}, u, and v satisfy, in plane stress concept, the following eight equations.

Equations of equilibrium (no body forces),

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \quad ,$$

Stress-Strain Relations,

$$\epsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\sigma_{yy}\right) + \alpha T$$

$$\epsilon_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu\sigma_{xx}\right) + \alpha T$$

$$\epsilon_{xy} = \frac{1}{2}\gamma_{xy} = \frac{1}{2G}\sigma_{xy} \quad ,$$

Strain-Displacement relations,

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \;;\; \epsilon_{yy} = \frac{\partial v}{\partial y} \;;\; \epsilon_{xy} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad ,$$

and in the case of plane stress,

$$\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0$$

$$\epsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) + \alpha T \quad .$$

2.0.2 Plane Strain Formulation.

In the case of plane strain defined by equations

$$u = u(x, y)$$

$$v = v(x, y)$$

$$w = 0$$

replace E, ν, and α of the stress-strain relations of plane stress formulation by E_1, ν_1, and α_1, respectively, where $E_1 = \frac{E}{1-\nu^2}$; $\nu_1 = \frac{\nu}{1-\nu}$; and $\alpha_1 = \alpha(1+\nu)$. The equations of equilibrium and strain-displacement relations remain unchanged.

2.0.3 Stress Formulation.

The solution of three partial differential equations satisfying the given boundary condition gives the stress distribution, σ_{xx}, σ_{xy}, and σ_{yy} in the body. The equilibrium equations are

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + X = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + Y = 0$$

and the compatibility condition is, for a simply connected body

$$\nabla^2 (\sigma_{xx} + \sigma_{yy} + \alpha ET) + (1+\nu)\left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}\right) = 0 \quad .$$

2.0.3.1 Solution of Airy's Stress Function.

I. Plane Stress.

For simply connected regions in the absence of the body forces, X, Y, the solution of this problem is simplified considerably by using Airy's stress function $\Phi(x, y)$. (See Section A1.3.6) Then

$$\sigma_{xx} = \frac{\partial^2 \Phi}{\partial y^2} \quad ; \quad \sigma_{yy} = \frac{\partial^2 \Phi}{\partial x^2} \quad ; \quad \sigma_{xy} = -\frac{\partial^2 \Phi}{\partial x \partial y} \quad .$$

The relations above satisfy the equilibrium equations identically, and substitution of these relations into the stress compatibility equation yields

$$\nabla^4 \Phi + \alpha E \; \nabla^2 T = 0 \quad ,$$

where

$$\nabla^4 \Phi = \nabla^2(\nabla^2 \Phi) = \frac{\partial^2 \Phi}{\partial x^4} + \frac{2 \, \partial^2 \Phi}{\partial x \partial y} + \frac{\partial^4 \Phi}{\partial y^4} \quad .$$

For this problem the boundary conditions should be expressed in terms of the stress function Φ.

II. Plane Strain.

For plane strain problems the governing equation can be obtained from those above by substituting E_1 and α_1 for E and α respectively, where

$E_1 = \frac{E}{1 - \gamma^2}$; $\alpha_1 = \alpha(1 + \nu)$.

$$\nabla^4 \Phi + \frac{\alpha E}{1 - \nu^2} \nabla^2 T = 0$$