3.2 CONICAL SHELLS.

This section recommends practices for predicting buckling of uniform stiffened and unstiffened circular conical shells under various types of static loading and suggests procedures that yield estimates of static buckling loads which are considered to be conservative.

Many studies have been conducted of the buckling of conical shells under various loading conditions. Knowledge of the elastic stability of conical shells, however, is not as extensive as that of cylindrical shells. Whereas the behavior of the two types of shells appears to be similar, significant differences in experimental results remain unexplained. Frequently, there are insufficient data to cover the wide range of conical-shell geometric parameters. In addition, some important loading cases and the effects of edge conditions remain to be studied. Some of these problems can be treated by digital computers. One such program is given in References 1 and 2.

3.2.1 ISOTROPIC CONICAL SHELLS.

The following are the recommended design procedures for isotropic conical shells under axial compression, bending, uniform hydrostatic pressure, and torsion, along with those for combined loads.

3.2.1.1 Axial Compression.

For conical shells under axial compression, there is considerable disagreement between experimental loads and the loads predicted by theory. These discrepancies have been attributed to the effects of imperfections of the structure and of edge-support conditions different from those assumed in the analysis, as well as to shortcomings of the small-deflection theory used.

A theoretical analysis [3] indicates that the critical axial load for long conical shells can be expressed as

$$P_{cr} = \gamma \frac{2\pi Et^2 \cos^2 \alpha}{\sqrt{3(1-\mu^2)}} \tag{1}$$

with the theoretical value of γ equal to unity. Experiments [4,5] indicate that within the range of the geometries of the tested specimens, there is no apparent effect of conical-shell geometry on the correlation factor. Therefore, γ can be taken as a constant. At present, it is recommended that γ be taken as the constant value.

$$\gamma = 0.33 \qquad (10\ \text{deg} < \alpha < 75\ \text{deg}) \quad , \tag{2}$$

which gives a lower bound to the experimental data. Buckling-load coefficients for cone semivertex angles greater than 75 deg must be verified by experiment because data are not available in this range. For $\alpha < 10$ deg the buckling load coefficient can be taken as that of a cylindrical shell having the same wall thickness as the cone and a length and radius equal to the slant height and average radius of curvature of the cone, respectively.

No studies have been published on the compressive buckling of conical shells in the yield region. Because the nominal stress level in a conical shell varies along its length, the effects of plasticity in conical shells are likely to differ from those in cylindrical shells. A conservative estimate of plasticity effects in conical shells could be obtained, however, if the reduction factors for cylindrical shells are used (Paragraph 3.1.1.1). The secant and tangent moduli should correspond to the maximum membrane compressive stress

$$\sigma_{max} = \frac{P}{2\pi \rho_1 t \cos^2 \alpha} \tag{3}$$

Figure 3.2-1 is an alignment chart devised to determine the critical axial force (P_{cr}) from equation (1) where the shell thickness (t) and the semivertex angle (α) are known. This nomograph is applicable in the elastic range for aluminum alloy.

From equations (1) and (3) the maximum membrane compressive stress is

$$\sigma_{cr} = \gamma E \frac{t}{r_1} \cos \alpha \quad . \tag{4}$$

Figure 3.2-2a is a nomograph of equation (4) to determine the critical axial stress when the shell thickness, small radius, and semivertex angle are known. The following example shows the use of the nomograph. A conical shell has a thickness (t) of 0.06 in., a small radius (r_1) of 40 in., and a semivertex angle of 60 deg. Determine the critical compressive stress resulting from an axial force. On the nomograph of Figure 3.2-2 join 40 on the r_1 scale with 0.06 on the t scale and extend the line until it meets line QR. From this point draw a line to 60 deg on the scale. This line intersects the σ_{cr} scale at 2600 psi, which is the critical buckling stress.

Figure 3.2-2b is useful if the stress falls into the plastic range for the three materials shown.

3.2.1.2 Bending.

For conical shells in bending, buckling occurs when the maximum compressive stress at the small end of the cone is equal to the critical compressive stress of a cylinder having the same wall thickness and the same local radius of curvature. The buckling moment is given by

$$M_{cr} = \frac{\gamma \pi E t^2 r_1 \cos^2 \alpha}{[3(1-\mu^2)]^{1/2}} \tag{5}$$

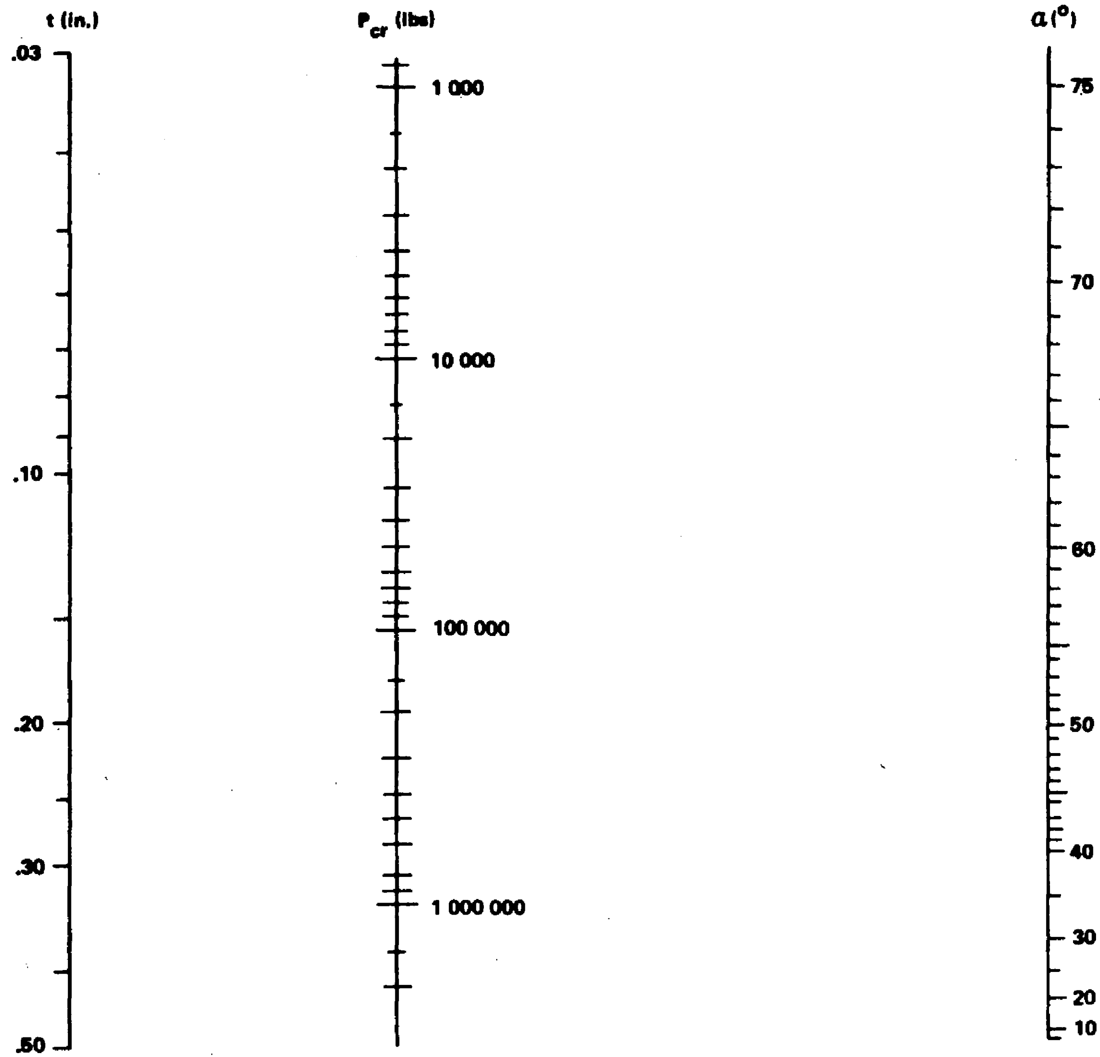


FIGURE 3.2-1. CRITICAL AXIAL LOAD FOR LONG CONICAL SHELL (ALUMINUM ALLOY MATERIAL), $E = 10.4 \times 10^6$ psi

with the theoretical value of γ equal to unity. Based on experimental data [6] it is recommended that the coefficient γ be taken as the constant value,

$$\gamma = 0.41 \qquad (10 \text{ deg} < \alpha < 60 \text{ deg}) \quad . \tag{6}$$

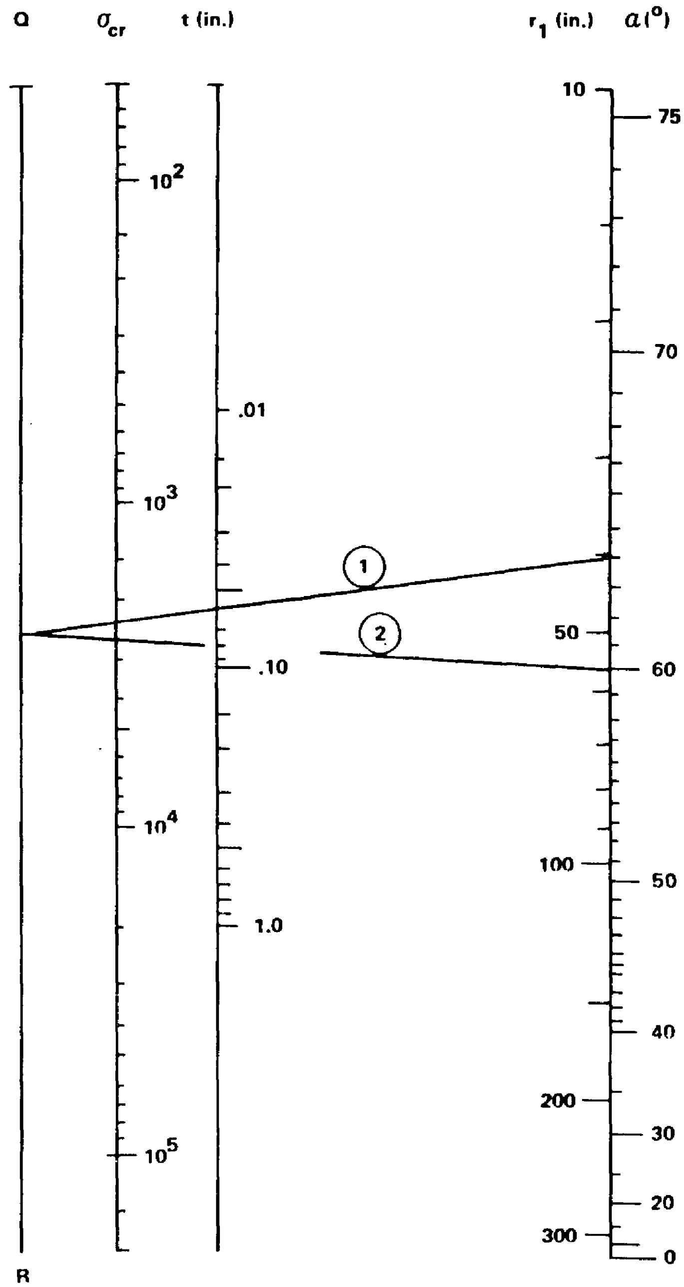


FIGURE 3.2-2a. CRITICAL COMPRESSIVE STRESS FOR LONG CONICAL SHELL (ALUMINUM ALLOY MATERIAL), $E = 10.4 \times 10^6$ psi

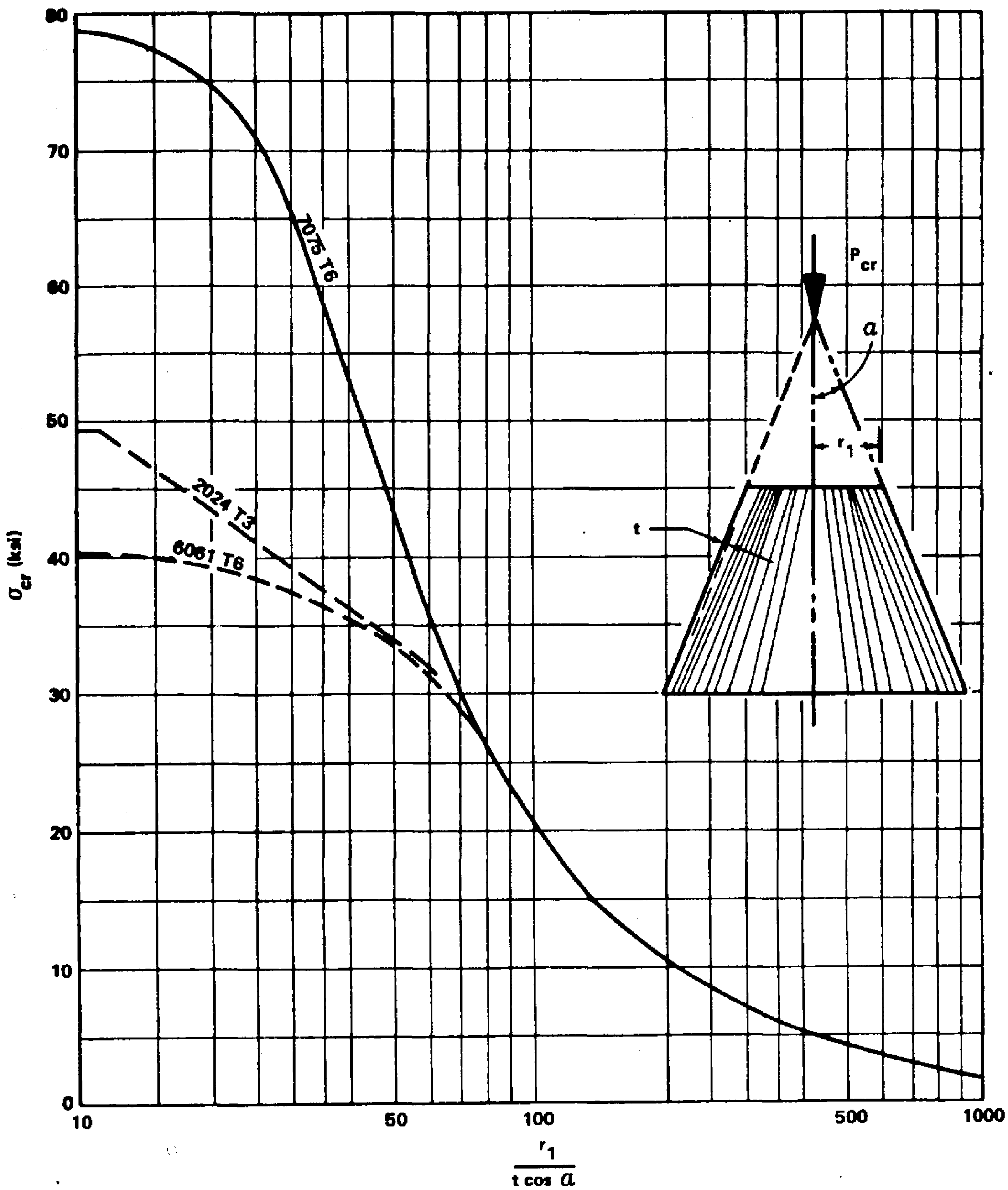


FIGURE 3.2-2b. BUCKLING OF ISOTROPIC CONICAL SHELL UNDER AXIAL COMPRESSION

Buckling load coefficients for cone semivertex angles greater than 60 deg must be verified by test. Buckling coefficients for equivalent cylindrical shells in bending can be used with semivertex angles less than 10 deg. For conical shells subjected to plastic stresses, the correction suggested for conical shells in axial compression may be used.

The buckling moment M_{cr} may be obtained from the nomograph of Figure 3.2-3, when the shell thickness t, the small radius r_1, and the semivertex angle α are known. The lines 1 and 2 on Figure 3.2-3 show the proper sequence of parameter alignment. This illustration can also be used for design purposes when a moment is given and a required thickness is needed.

3.2.1.3 Uniform Hydrostatic Pressure.

The theoretical buckling pressure of a conical shell which buckles into several circumferential waves $(n > 2)$ can be expressed [7] in the approximate form

$$p_{cr} = \frac{0.92\,E\,\gamma}{\left(\frac{L}{\bar{\rho}}\right)\left(\frac{\bar{\rho}}{t}\right)^{5/2}} \qquad (7)$$

Experiments [8, 9] show a relatively wide scatter band for the value of γ but indicate that the constant value

$$\gamma = 0.75 \qquad (8)$$

should provide a lower bound for the available data. Figure 3.2-4 gives the solution to equation (7) for values of $\bar{\rho}/t$ and $L/\bar{\rho}$. The curves are applicable in the elastic range only.

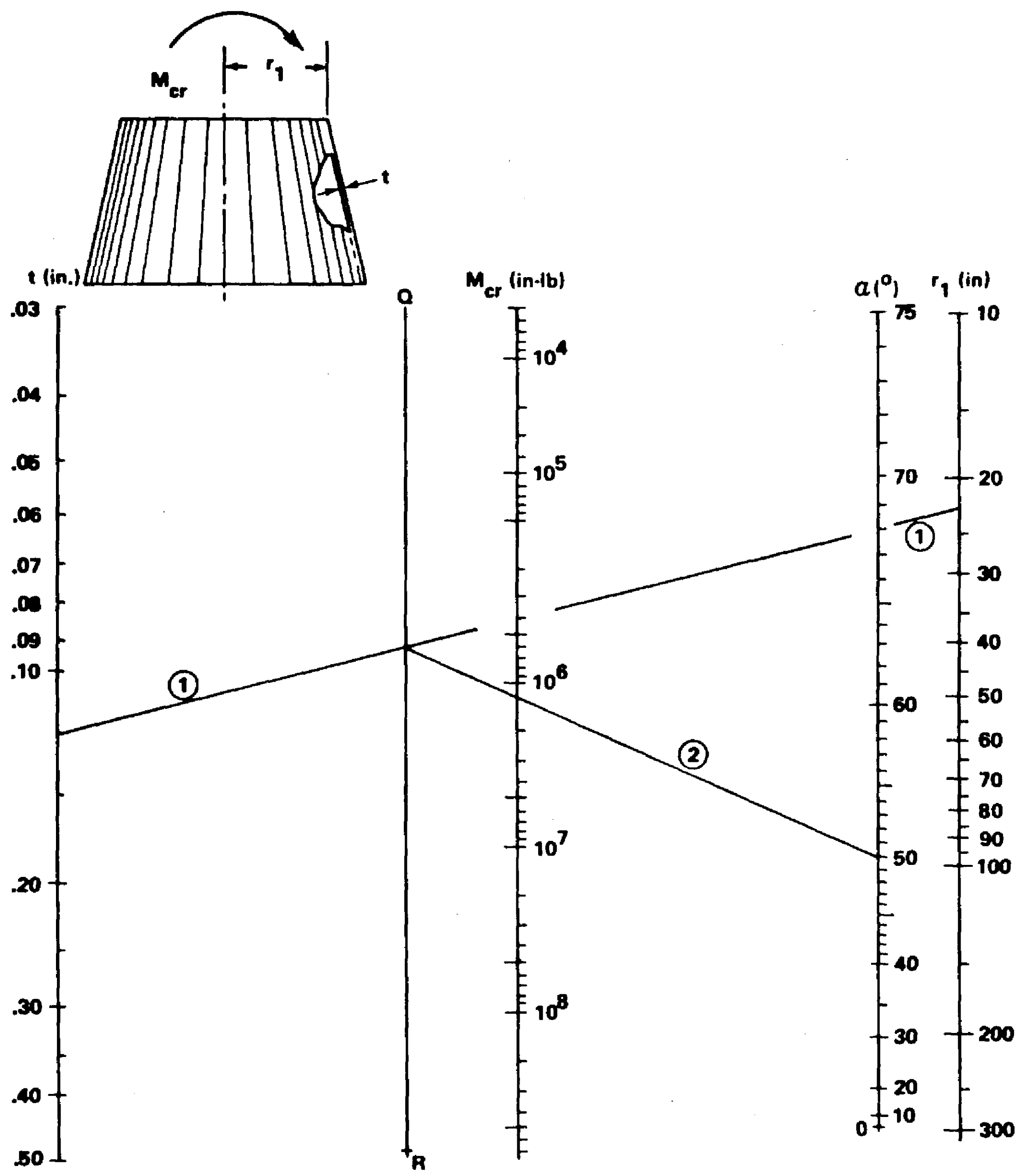


FIGURE 3.2-3. BUCKLING MOMENT FOR CONICAL TRUNCATED SHELL (ALUMINUM ALLOY MATERIAL), $E = 10.4 \times 10^6$ psi

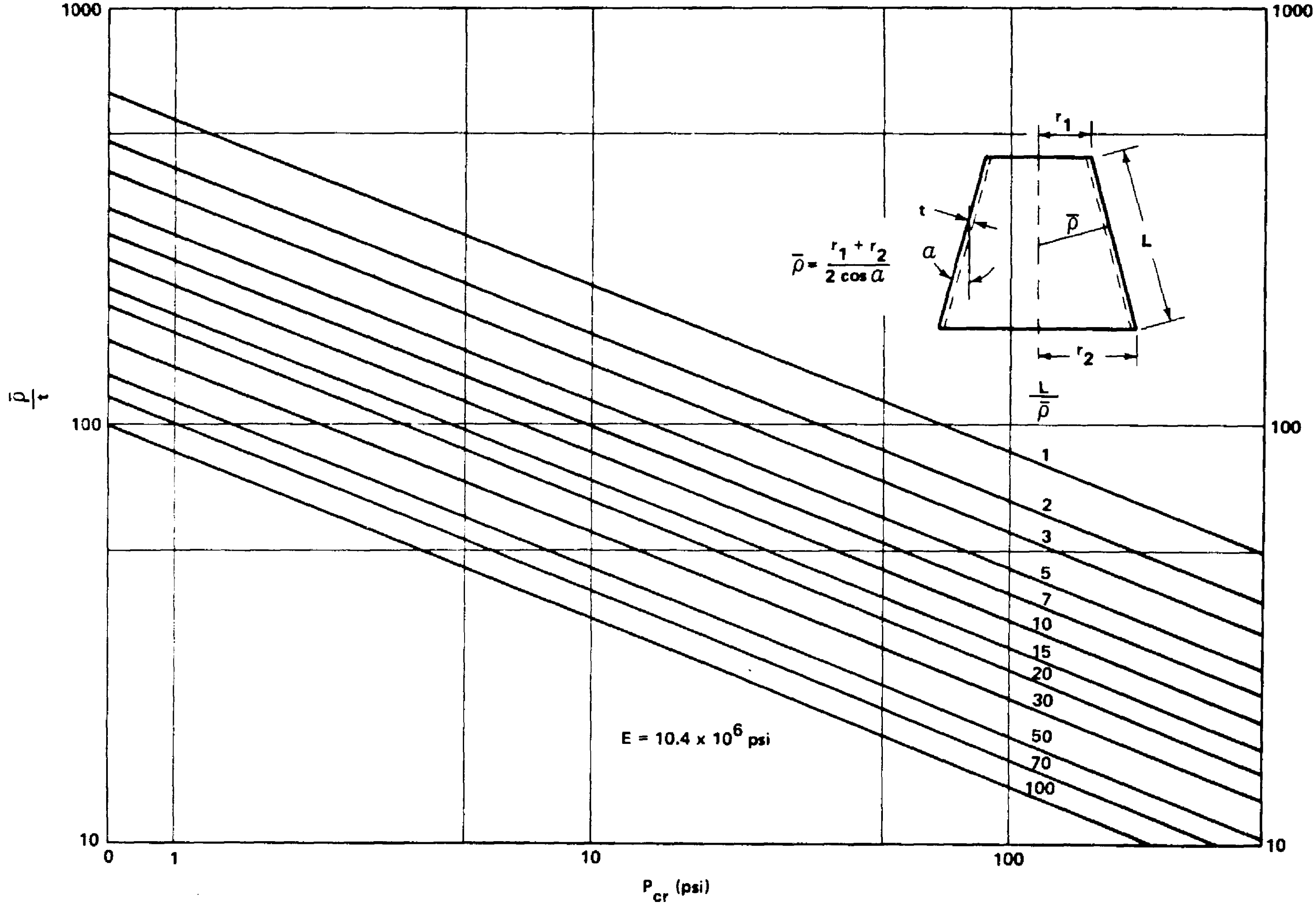


FIGURE 3.2-4. BUCKLING ALLOWABLE PRESSURES OF ISOTROPIC CONICAL SHELL UNDER UNIFORM EXTERNAL HYDROSTATIC PRESSURE

For conical shells which buckle in the plastic range, the plasticity correction for moderate-length cylindrical shells may be used for the range of the conical shell geometries considered. The procedure is to use the E_1 curves in Paragraph 3.1.6. The moduli should correspond to the maximum circumferential compressive stress at the large end of the conical shell:

$$\sigma_{max} = p_{cr} (\rho_2/t) \quad . \tag{9}$$

3.2.1.4 Torsion.

An approximate equation for the critical torque of a conical shell [10] is

$$T_{cr} = 52.8 \gamma D \left(\frac{t}{\ell}\right)^{1/2} \left(\frac{r}{t}\right)^{5/4} \tag{10}$$

where

$$r = r_2 \cos \alpha \left\{1 + \left[1/2\left(1 + \frac{r_2}{r_1}\right)\right]^{1/2} - \left[1/2\left(1 + \frac{r_2}{r_1}\right)\right]^{-1/2}\right\} \frac{r_1}{r_2} \quad . \tag{11}$$

The variation of $r/r_2 \cos \alpha$ with r_1/r_2 is shown in Figure 3.2-5. For design purposes it is recommended that the torsional-moment coefficient in equation (10) be taken as the constant value

$$\gamma = 0.67 \quad . \tag{12}$$

Figure 3.2-6 is a nomograph devised to determine the buckling torque of a conical shell (equation 10) when t, t/ℓ, and r/t are known.

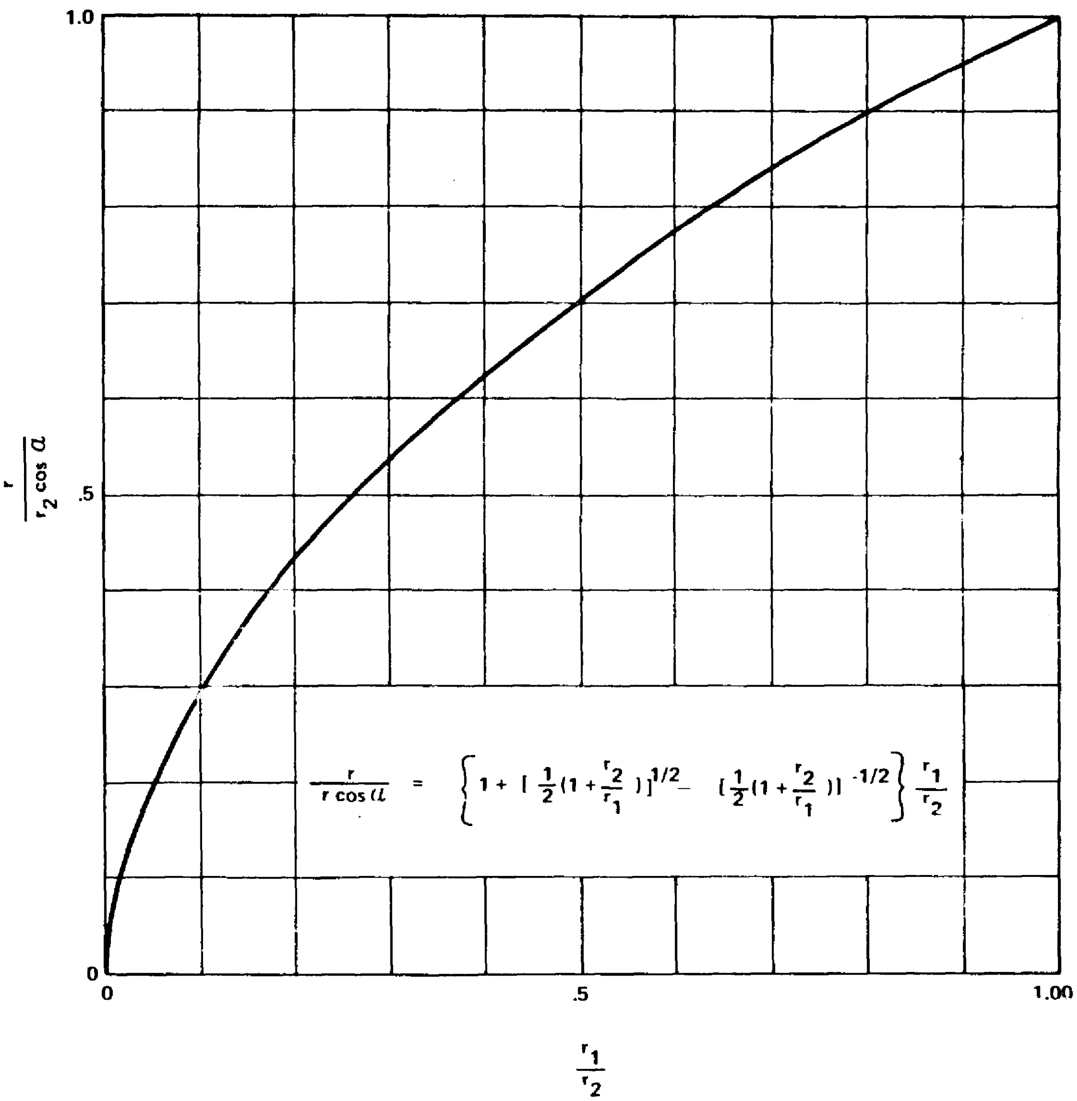
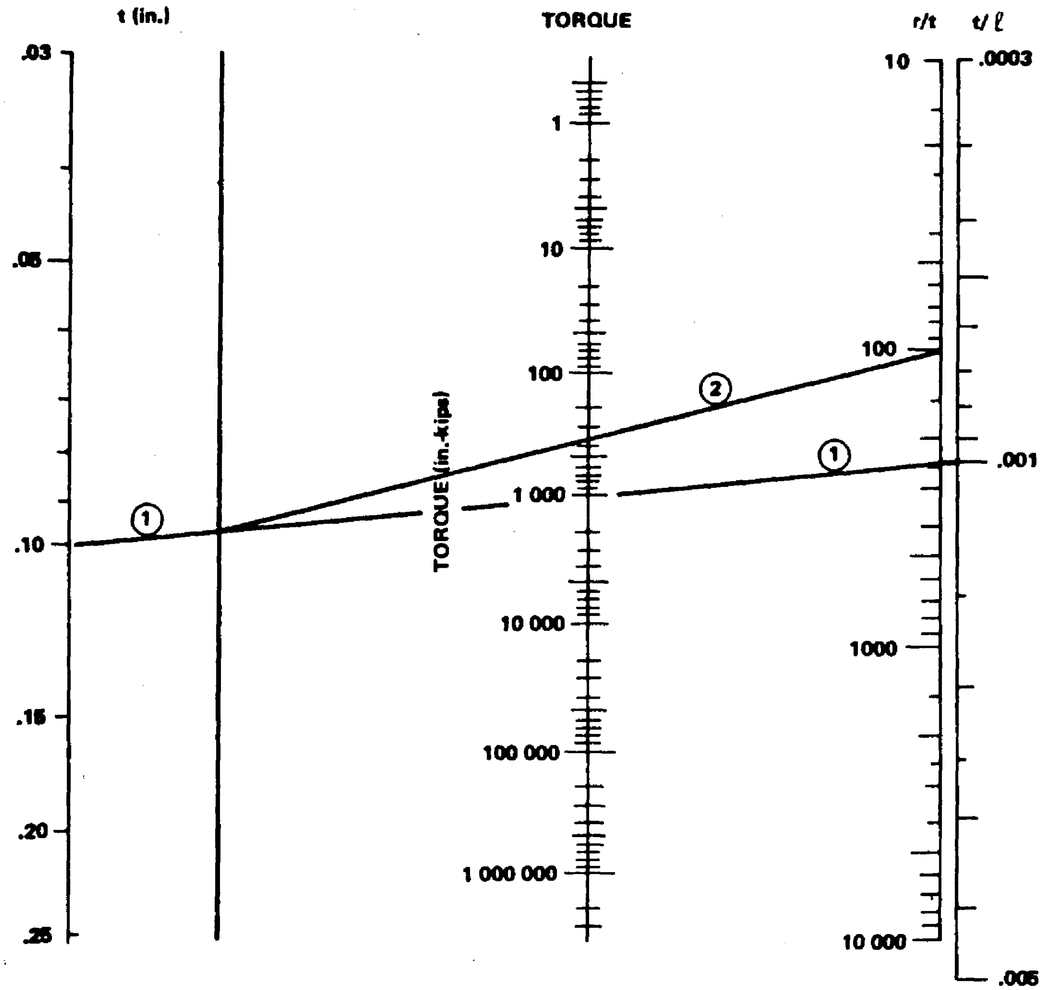


FIGURE 3.2-5. VARIATION OF $r/r_2 \cos \alpha$ WITH r_1/r_2

No data are available for the plastic buckling of conical shells in torsion. The plasticity factor used for cylindrical shells in torsion should, however, give conservative results. The secant modulus should correspond to the maximum shear stress at the small end of the cone, given by

$$\tau_{cr} = \frac{T_{cr}}{2\pi r_1^2 t} \qquad (13)$$



NOTE: SEE FIGURE 3.2-5 TO DETERMINE "r".

FIGURE 3.2-6. ALLOWABLE TORQUE FOR CONICAL SHELL
$(E = 10.4 \times 10^6)$

3.2.1.5 Combined Loads.

I. Pressurized Conical Shells in Axial Compression.

The theory for predicting buckling of internally pressurized conical shells under axial compression [11] differs from that for cylindrical shells in two respects. First, the axial load-carrying capacity is a function of internal pressure and exceeds the sum of the load-carrying capacity of the unpressurized shell and the pressure load at the small end of the cone. Second, results of analyses for conical shells indicate that edge conditions at the small end have a significant effect on the axial load-carrying capacity. The results are independent of edge conditions at the large end for long cones.

There are insufficient data to warrant design use of the entire increase in load-carrying capacity of internally pressurized conical shells. It is therefore recommended that the critical axial compressive load for a pressurized conical shell be determined by adding the pressurization load $\pi r_1^2 p$ at the small end of the cone to the compressive buckling load at the conical shell. Then

$$p_{cr} = \left[\frac{\gamma}{\sqrt{3(1-\mu^2)}} + \Delta\gamma \right] (2\pi Et^2 \cos^2 \alpha) + \pi r_1^2 p \quad . \qquad (14)$$

The unpressurized compressive-buckling coefficient γ is equal to 0.33, and the increase in the buckling coefficient $\Delta\gamma$ for the equivalent cylindrical shell is given in Figure 3.2-7. The critical axial load may be increased above the value given by equation (14), however, if the increase is justified by test.

II. Pressurized Conical Shells in Bending.

As in the case of unpressurized conical shells subjected to pure bending, no theory has yet been developed for pressurized conical shells under bending. For conservative design, therefore, the design moment of the pressurized conical shell is written as

$$M_{press} = \left[\frac{\gamma}{\sqrt{3 \ 1-\mu^2}} + \Delta\gamma \right] \pi E r_1 (t \cos \alpha)^2 + \frac{p \pi r_1^3}{2} \quad . \qquad (15)$$

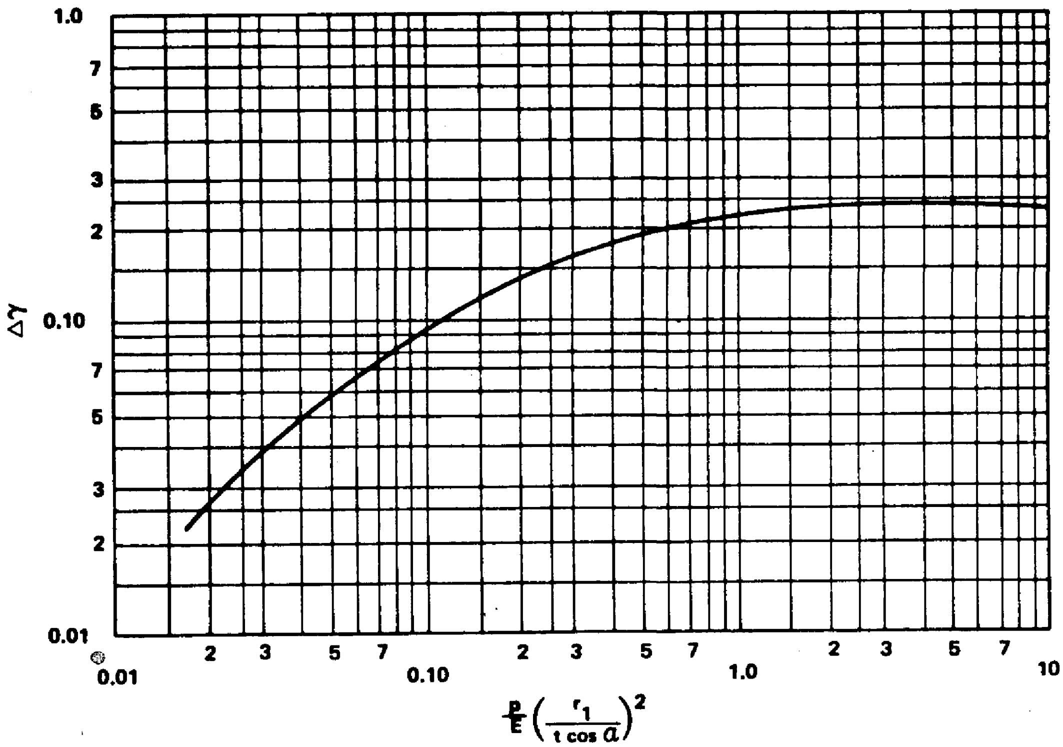


FIGURE 3. 2-7. INCREASE IN AXIAL COMPRESSIVE BUCKLING-STRESS COEFFICIENTS OF CONICAL SHELLS RESULTING FROM INTERNAL PRESSURE

The unpressurized compressive-buckling coefficient γ is equal to 0. 41, and the increase in buckling coefficient $\Delta\gamma$ for the equivalent cylindrical shell can be obtained from Figure 3. 2-7.

III. Combined Axial Compression and Bending for Unpressurized and Pressurized Conical Shells.

Some experimental interaction curves have been obtained for unpressurized and pressurized conioal shells under combined axial compression and bending [6]. These investigations indicate that the following straight-line interaction curve for conical shells is adequate for design purposes:

$$R_c + R_b = 1 \qquad (16)$$

where

$$R_c = \frac{P}{P_{cr}} \qquad (17)$$

and

$$R_b = \frac{M}{M_{cr}} \quad . \qquad (18)$$

For equations (17) and (18),

P = applied compressive load.

P_{cr} = critical compressive load for cone not subjected to bending, obtained from equations (1) and (2) for unpressurized shells, and from equation (14) for pressurized shells.

M = applied bending moment.

M_{cr} = critical moment for cone not subjected to axial compression, as obtained from equations (5) and (6) for unpressurized shells, and from equation (15) for pressurized shells.

If actual test values of P_{cr} and M_{cr} are used, the straight-line interaction curve may no longer be conservative, and the entire interaction curve must be substantiated by test.

IV. Combined External Pressure and Axial Compression.

For a conical shell subjected to combined external pressure and axial compression, the relationship

$$R_c + R_p = 1 \tag{19}$$

is recommended for design purposes. Where

$$R_p = \frac{P}{P_{cr}} \tag{20}$$

P_{cr} is given by equation (7) and (8), and R_c is given by equation (17).

V. Combined Torsion and External Pressure or Axial Compression.

For conical shells under combined torsion and external hydrostatic pressure the following interaction formula is recommended for design purpose:

$$R_t + R_p = 1 \tag{21}$$

with

$$R_t = \frac{T}{T_{cr}} \tag{22}$$

where T_{cr} is given by equations (10), (11), and (12), and R_p is given by equation (20).

For conical shells under combined torsion and axial compression, the following interaction formula is recommended for design purposes:

$$R_t + R_c = 1 \tag{23}$$

where R_t is given by equation (22) and R_c by equation (17).

3.2.2 ORTHOTROPIC CONICAL SHELLS.

The theory of buckling of orthotropic conical shells is valuable in determining adequate buckling criteria for shells which are geometrically orthotropic because of closely spaced meridional or circumferential stiffening, as well as for shells constructed of a material whose properties differ in the two directions. An extension of the Donnell-type isotropic conical shell theory to conical shells with material orthotropy is given in Reference 12, whereas buckling of conical shells with geometric orthotropy is considered in Reference 13. Numerical results are limited to only a few values of the many parameters, but these provide the basis for tentative generalizations. Few experiments have been conducted. Following are the design recommendations based on the limited data available. The computer programs discussed in Subsection 3.4 are also recommended.

3.2.2.1 Uniform Hydrostatic Pressure.

I. Constant-Thickness Orthotropic Material.

A limited investigation [14] indicates that the relationship between the theoretical buckling pressures of an orthotropic conical shell and of the so-called equivalent orthotropic cylinder is similar to that between the buckling pressures of an isotropic conical shell and of the equivalent isotropic cylinder. In both cases the equivalent cylinder is defined as one having a length equal to the slant length, L, of the conical shell; a radius equal to the average radius of curvature, $\bar{\rho}$, of the conical shell, and the same thickness. Thus, the theoretical hydrostatic buckling pressures for supported moderate-length orthotropic conical shells [15, 16] can be expressed as

$$P_{cr} = \frac{0.86\,\gamma}{\left(1 - \mu_s \mu_\theta\right)^{3/4}} E_s^{1/4} E_\theta^{3/4} \left(\frac{\bar{\rho}}{L}\right) \left(\frac{t}{\bar{\rho}}\right)^{5/2} , \qquad (24)$$

which reduces to the corresponding expression for the isotropic cone when

$$E_s = E_\theta = E \quad ,$$
$$\mu_s = \mu_\theta = \mu \quad . \tag{25}$$

Only limited experimental data exist for conical shells constructed for an orthotropic material [17]. In the absence of a more extensive range of test results, it is recommended that the value of the correlation coefficient γ be taken as 0.75 for both orthotropic and isotropic cones.

II. Stiffened Conical Shells.

The stability of conical shells stiffened by rings under uniform hydrostatic pressure has also been investigated [13, 18]. In these investigations, all rings were assumed to have the same cross-sectional shape and area but could have variable spacing. The approximate buckling formulas given in these references are not recommended for use in design until a larger amount of substantiating test data becomes available.

3.2.2.2 Torsion.

I. Constant-Thickness Orthotropic Material.

The investigation reported in Reference 19 indicates that the theoretical buckling torque of an orthotropic conical shell is approximated by that of an equivalent orthotropic cylinder having a length equal to the height, ℓ, of the conical shell and having the same thickness and radius given in equation (11). Refer to Figure 3.2-5 for the variation of $\frac{r}{r_2 \cos\alpha}$ with $\frac{r_1}{r_2}$.

The critical torque of a moderate-length orthotropic conical shell may then be approximated by the expression

$$T_{cr} = 4.57\,\gamma \frac{E_\theta^{5/8} E_s^{3/8} r^2 t}{\left(1 - \mu_\theta \mu_s\right)^{5/8}} \left(\frac{t}{r}\right)^{5/4} \left(\frac{r}{\ell}\right)^{1/2} \quad . \tag{26}$$

A reduction factor of $\gamma = 0.67$ (the value given for isotropic conical shells) is recommended. The few data points available for fiberglass-reinforced epoxy conical shells [17] yield a larger value of γ but fall within the scatter band for the isotropic shell of constant thickness.

II. Ring-Stiffened Conical Shells.

Although no accurate theoretical calculations have been made for ring-stiffened conical shells in torsion, a few tests [17] indicate that when the rings are equally spaced and have the same cross-sectional shape and area, a procedure similar to that for the materially orthotropic conical shell will yield adequate results. The critical torque of such a ring-stiffened conical shell may thus be approximated by the critical torque of a ring-stiffened cylinder having the radius, length, and thickness described above. The critical torque of a ring-stiffened cone with uniformly spaced rings is then given by

$$T_{cr} = 4.57\,\gamma \frac{E\, r^2 t}{\left(1 - \mu_s \mu_\theta\right)^{5/8}} \left(\frac{t}{r}\right)^{5/4} \left(\frac{r}{\ell}\right)^{1/2} \left(1 + \eta_0\right)^{5/8} \tag{27}$$

where (Fig. 3.2-8)

$$\eta_o = 12(1-\mu^2)\,\frac{E_r}{E}\left[\frac{I_r}{L_o t^3} + \frac{A_r}{L_o t}\left(\frac{\tilde{z}_r - e_r}{t}\right)^2\right] + 12\left(\frac{e_r}{t}\right)^2 \tag{28}$$

and the factor γ is recommended to be taken equal to 0.67. The few available test results also indicate a larger value of γ, but these again fall within the scatter band for the isotropic conical shell of constant thickness.

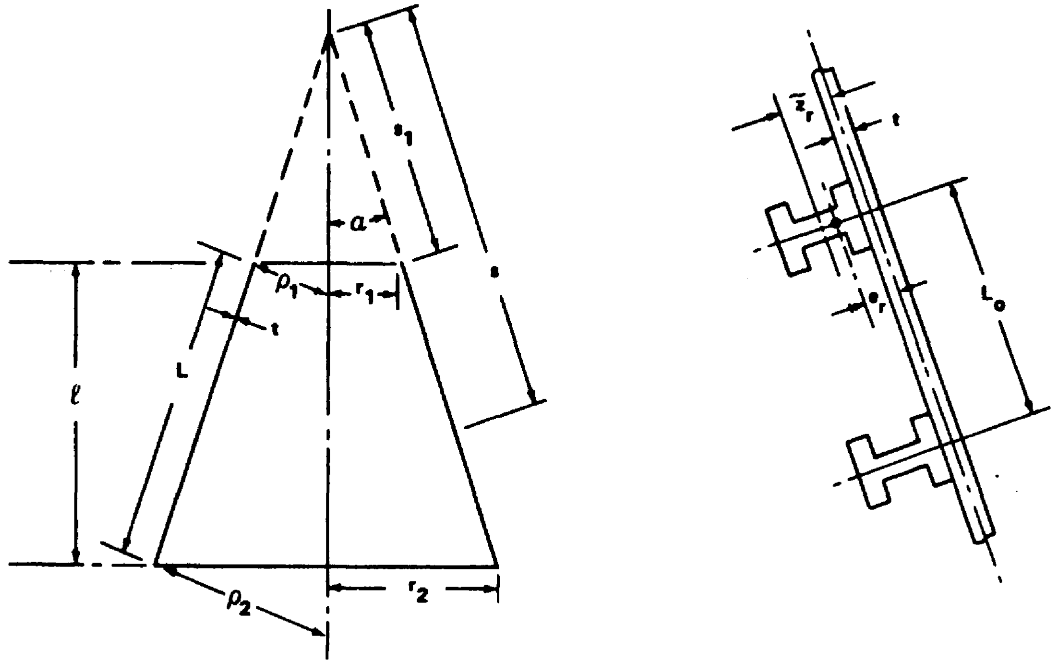


FIGURE 3.2-8. NOTATION FOR RING-STIFFENED CONICAL SHELLS

3.2.3 SANDWICH CONICAL SHELLS.

If the sandwich core is resistant to transverse shear so that its shear stiffness can be assumed to be infinite, the previous results for isotropic and orthotropic conical shells may readily be adapted to the analysis of sandwich conical shells by the following method.

3.2.3.1 Isotropic Face Sheets.

If the core is assumed to have infinite transverse shear stiffness and no load-carrying capacity in the meridional or circumferential directions, the analysis for isotropic conical shells of constant thickness may be used for isotropic sandwich conical shells of constant thickness. An equivalent modulus and thickness must be defined for the sandwich shell. The face sheets may be of different thicknesses and of different materials, subject to the restriction that the Poisson's ratios of the two materials be identical. If the stretching and bending stiffnesses of such an isotropic sandwich shell are equated to the stretching and bending stiffnesses of an equivalent constant-thickness isotropic shell having the same neutral surface dimensions, then

$$\overline{E}\ \overline{t} = E_1 t_1 + E_2 t_2 \quad . \tag{29a}$$

$$\frac{\overline{E}\,(\overline{t})^3}{12} = \frac{h^2}{\dfrac{1}{E_1 t_1} + \dfrac{1}{E_2 t_2}} \quad . \tag{29b}$$

Then the modulus and the thickness of the equivalent constant-thickness isotropic shell are

$$\overline{t} = \frac{h\sqrt{12}}{\sqrt{\dfrac{E_1 t_1}{E_2 t_2}} + \sqrt{\dfrac{E_2 t_2}{E_1 t_1}}} \quad . \tag{30a}$$

$$\overline{E} = \frac{E_1 t_1 + E_2 t_2}{\overline{t}} \quad . \tag{30b}$$

The buckling loads of the isotropic sandwich shell may now be taken as the buckling loads of the equivalent isotropic shell of constant thickness as listed below.

Load	Paragraph Reference
Axial Compression	3.2.1.1
Bending	3.2.1.2
Uniform hydrostatic pressure	3.2.1.3

Load	Paragraph Reference
Torsion	3.2.1.4
Pressurized conical shells in axial compression	3.2.1.5-I
Pressurized conical shells in bending	3.2.1.5-II
Combined axial compression and bending for unpressurized and pressurized conical shells	3.2.1.5-III
Combined external pressure and axial compression	3.2.1.5-IV
Combined torsion and external pressure or axial compression	3.2.1.5-V

In the absence of experimental data, the reduction or correlation factors for isotropic shells of constant thickness are recommended for isotropic sandwich shells.

3.2.3.2 Orthotropic Face Sheets.

If the core is assumed to have infinite transverse shear stiffness and no load-carrying capacity in the meridional or circumferential directions, the available results for conical shells of constant-thickness orthotropic material may be used for sandwich conical shells having orthotropic faces. The face sheets may be of different thicknesses but of the same orthotropic material as long as their principal axes are oriented in the same direction. The same procedure as for sandwich shells having isotropic face sheets leads to the following thickness and material properties of the equivalent materially orthotropic conical shells of constant thickness:

$$\bar{t} = \frac{\sqrt{12}\, h}{\sqrt{\frac{t_1}{t_2}} + \sqrt{\frac{t_2}{t_1}}} \qquad (31a)$$

$$\frac{\bar{E}_s}{\bar{\bar{E}}_s} = \frac{\bar{E}_\theta}{\bar{\bar{E}}_\theta} = \frac{\bar{G}}{G} = \frac{t_1 + t_2}{\bar{t}} \quad . \tag{31b}$$

$$\frac{\bar{\mu}_s}{\mu_s} = \frac{\bar{\mu}_\theta}{\mu_\theta} = 1 \quad . \tag{31c}$$

The buckling load of the orthotropic sandwich conical shell is then the buckling load of the equivalent conical shell of orthotropic material having constant thickness. The reduction or correlation factors for isotropic shells of constant thickness are recommended for use for sandwich shells with orthotropic face sheets.

3.2.3.3 Local Failure.

Thus far, only overall buckling has been considered as a criterion of failure. Other modes of failure are possible, however. For honeycomb-core sandwich shells, failure may occur because of core crushing, intracell buckling, and face wrinkling. The use of relatively heavy cores ($\delta > 0.03$) will usually prevent core crushing. Lighter cores may prove to be justified as data become available. No studies have been conducted that predict localized buckling failures under stress states that are a function of position. If we assume, however, that the stress state varies only slightly over the buckled region, the following approximate equations developed for cylindrical shells can be used to predict failure from intracell buckling and face wrinkling of heavy honeycomb-core sandwich conical shells with equal-thickness face sheets under uniaxial loading. For intracell buckling

$$\sigma_s = 2.5\, E_R \left(\frac{t_f}{S}\right)^2 \tag{32}$$

where S is the core cell size expressed as the diameter of the largest inscribed circle and

$$E_R = \frac{4\,E_f\,E_{tan}}{\left(\sqrt{E_f} + \sqrt{E_{tan}}\right)^2} \tag{33}$$

where E_f and E_{tan} are the elastic and tangent moduli of the face-sheet material. If initial dimpling is to be checked, the equation

$$\sigma_s = 2.2\,E_R \left(\frac{t_f}{S}\right)^2 \tag{34}$$

should be used. The sandwich will still carry loads if initial dimpling occurs. For wrinkling

$$\sigma_s = 0.50\,(E_{sec}\,E_z\,G_{sz})^{1/3} \tag{35}$$

where E_z is the modulus of the core in a direction perpendicular to the plane of the core, and G_{sz} is the transverse shear modulus of the core. If biaxial compressive stresses are applied to the sandwich, the coefficients of equations must be reduced by the factor $(1 + f^3)^{-1/3}$, where f is the ratio of minimum to maximum principal compressive stress in the face sheets.

Wrinkling and intracell-buckling equations which consider strength of bond, strength of foundation, and initial waviness of the face sheets are given in References 20, 21, and 22.

The plasticity correction factor given for isotropic conical shells in axial compression may be applied also to isotropic sandwich conical shells. The factor is applicable to sandwich cones with stiff cores and becomes somewhat conservative as the shear stiffness of the core is decreased [23].