B7.2.2.0 LOCAL LOADS ON CYLINDRICAL SHELLS

This section presents a method to obtain cylindrical shell membrane and bending stresses at an attachment-to-shell juncture resulting from arbitrary loads induced through rigid attachments on pressurized or unpressurized shells. Shell geometry and loading conditions are used to obtain normal and bending stress resultants and deflections from a computer program for radial-type loads (P and M_a). Membrane shear stresses caused by shearing loads (V_a) and twisting moments (M_T) can be calculated directly without the computer program. Deflections are not calculated for shearing loads and twisting moments.

Local load stresses reduce rapidly at points removed from the attachment-to-shell juncture. Boundaries of that region of the shell influenced by the local loads can be determined for those load cases calculated with the computer program by investigation of the stresses and deflections at points removed from the attachment.

The additional stiffness of the shell caused by internal pressure (pressure coupling) is taken into account by the computer program for determination of local load stress resultants and deflections. The stress resultants induced in the shell by the internal pressure are not included in the computer program results and must be superimposed upon the local load stress resultants calculated by the method contained in this section.

B7.2.2.1 GENERAL

I NOTATION

The notations presented in this section are not applicable to the computer program. The computer program variables are defined in the Astronautics Computer Utilization Handbook.

a	-	fillet radius at attachment-to-shell juncture or longitudinal half diameter of elliptical load pad, in.
b	-	x-coordinate distance to center of attachment ($b = L/2$) or circumferential half diameter of elliptical load pad, in.
c	-	half length of square attachment, in.
c_1	-	longitudinal half length of rectangular attachment, in.
c_2	-	circumferential half length of rectangular attachment, in.
E	-	modulus of elasticity, psi
f_x	-	normal longitudinal stress, psi
f_y	-	normal circumferential stress, psi
f_{xy}	-	shear stress, psi
K_n, K_b	-	stress concentration parameters for normal stresses and bending stresses, respectively
L	-	length of cylinder
M_a	-	applied overturning moment, in.-lb.
M_T	-	applied twisting moment, in.-lb.
M_j	-	internal bending moment stress resultant per unit length of shell, in.-lb/in.
n	-	number of equally spaced attachments in the circumferential direction
N_j	-	internal normal force stress resultant per unit length of shell, lb/in.
p	-	uniform load intensity, psi
P	-	radial load or total distributed radial load, lb.
q	-	internal pressure, psi

B7.2.2.1 GENERAL (Concluded)

I NOTATION (Concluded)

r - radius of circular attachment, in.

R - radius of cylindrical shell, in.

s - circumferential arc length, in.

T - thickness of cylindrical shell, in.

u - longitudinal displacement, in.

v - circumferential displacement, in.

V_a - applied concentrated shear load or total distributed shear load, lb.

w - radial displacement, in.

x - longitudinal coordinate, in.

y - circumferential coordinate, in.

z - radial coordinate, in.

θ - polar coordinate

ν - Poisson's ratio

ϕ - circumferential cylindrical coordinate

Subscripts

a - applied ($a = 1$ or $a = 2$)
b - bending
i - inside
j - internal ($j = x$ or $j = y$)
m - mean (average of outside and inside)
n - normal
o - outside
x - longitudinal
y - circumferential
z - radial

1 - longitudinally directed applied load vector or longitudinal direction

2 - circumferentially directed applied load vector or circumferential direction

B7.2.2.1 GENERAL

II SIGN CONVENTION

Local loads applied at an attachment-to-shell induce a biaxial state of stress on the inside and outside surfaces of the shell. The longitudinal stress (f_x), circumferential stress (f_y), shear stress (f_{xy}), the positive directions of the applied loads (M_a*, M_T, P, q, and V_a), the stress resultants (M_j and N_j), and the positive directions of the displacements (u, v, and w) are indicated in Figure B7.2.2.1-1.

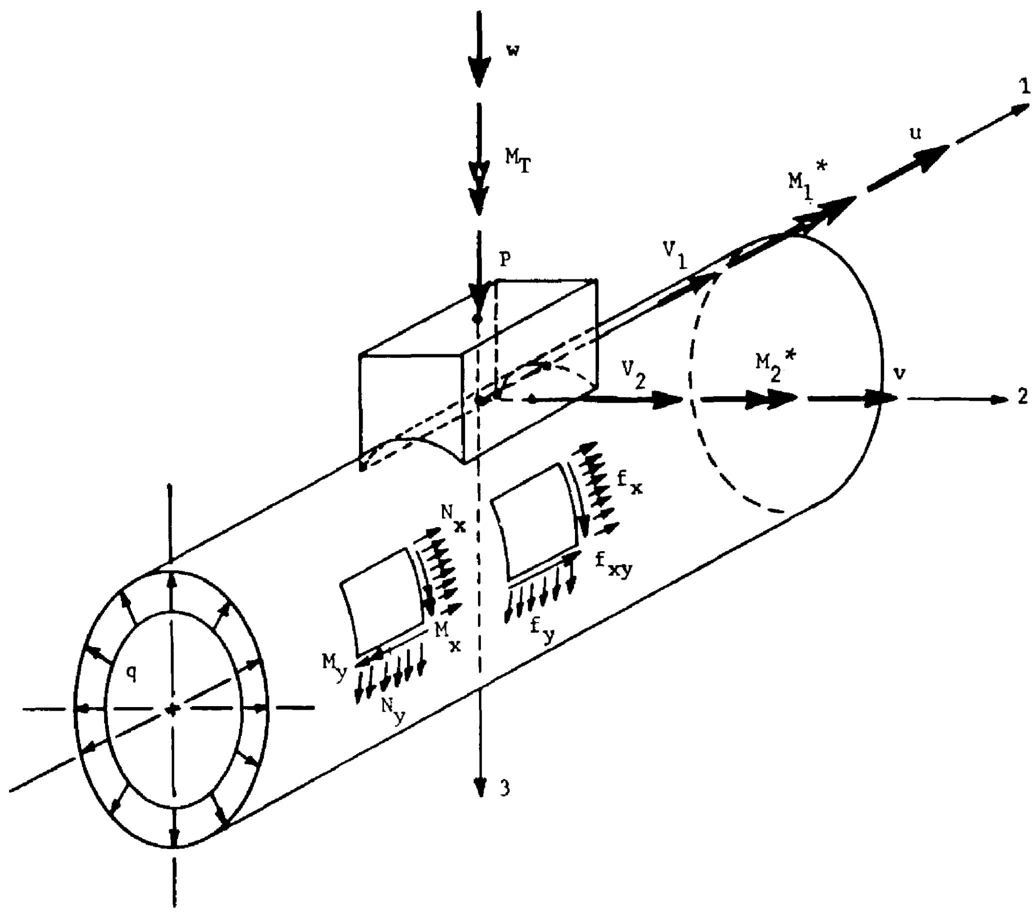

Fig. B7.2.2.1-1 Stresses, Stress Resultants, Loads, and Displacements

* The applied overturning moment M_1 (M_2) is represented by a longitudinally (circumferentially) directed vector but is defined as an applied circumferential (longitudinal) overturning moment since its effect is in the circumferential (longitudinal) direction.

The geometry of the shell and attachment, the local coordinate system at the attachment, and the coordinate system of the shell are indicated in Figure B7.2.2.1-2.

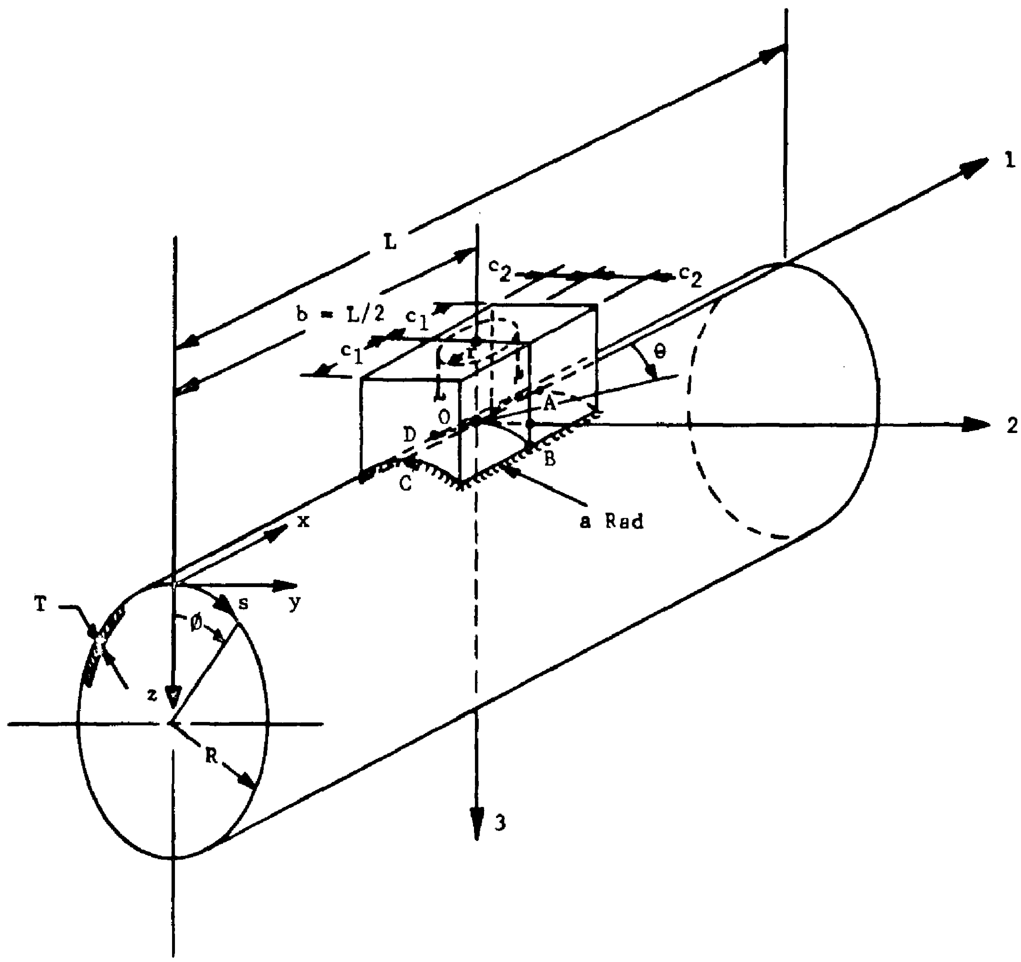

Fig. B7.2.2.1-2 Shell and Attachment Geometry

It is possible to predict the sign of the induced stress, tensile (+) or compressive (-), by considering the deflections of the shell resulting from various loading modes.

Mode I (radial load), Figure B7.2.2.1-3, shows a positive radial load (P) transmitted to the shell by a rigid attachment. The load (P) causes compressive membrane stresses and local bending stresses adjacent to the attachment. The compressive membrane stresses are similar to the stresses induced by an external pressure. The local bending stresses result in tensile bending stresses on the inside of the shell and compressive bending stresses on the outside of the shell at points A, B, C and D.

Modes II (circumferential moment) and III (longitudinal moment), Figure B7.2.2.1-3, show negative overturning moments (M_a) transmitted to the shell by rigid attachments. The overturning moments (M_a) cause compressive and tensile membrane stresses and local bending stresses adjacent to the attachment. Tensile membrane stresses are induced in the shell at B or C, similar to the stresses caused by an internal pressure. Compressive membrane stresses are induced in the shell at D or A, similar to the stresses caused by an external pressure. The local bending stresses cause tensile bending stresses in the shell at B or C on the outside and at D or A on the inside, and cause compressive bending stresses in the shell at B or C on the inside and at D or A on the outside.

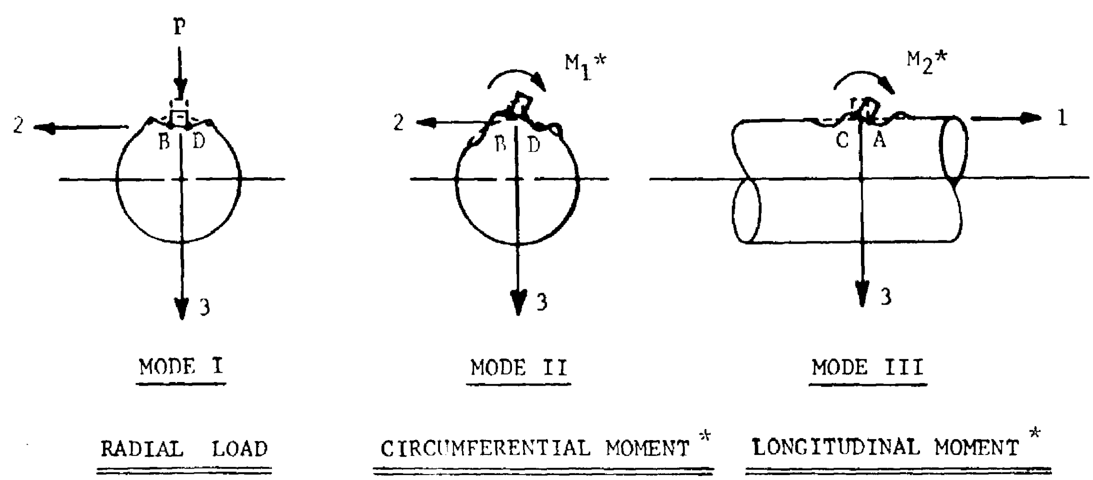

Fig. B7.2.2.1-3 Loading Modes

*The applied overturning moment M_1 (M_2) is represented by a longitudinally (circumferentially) directed vector but is defined as an applied circumferential (longitudinal) overturning moment since its effect is in the circumferential (longitudinal) direction.

The signs of the stresses induced in the shell adjacent to the attachment by positive applied loads for rigid attachments are shown in Figure B7. 2. 2. 2-1 "Stress Calculation Sheet". The figure or parts thereof can be reproduced and used as calculation sheets.

B7.2.2.1 GENERAL

III LIMITATIONS OF ANALYSIS

Considerable judgment must be used in the interpretation of the results of this section and in the establishment of the geometry and loadings used in the analysis.

Six general areas must be considered for limitations: attachment and shell size, attachment location, shift in maximum stress location, stresses caused by shear loads, geometry and loading.

A Size of Attachment with Respect to Shell Size

The analysis is applicable to small attachments relative to the shell size and to thin shells. The limitations on these conditions are shown by the shaded area of Figure B7.2.2.1-4.

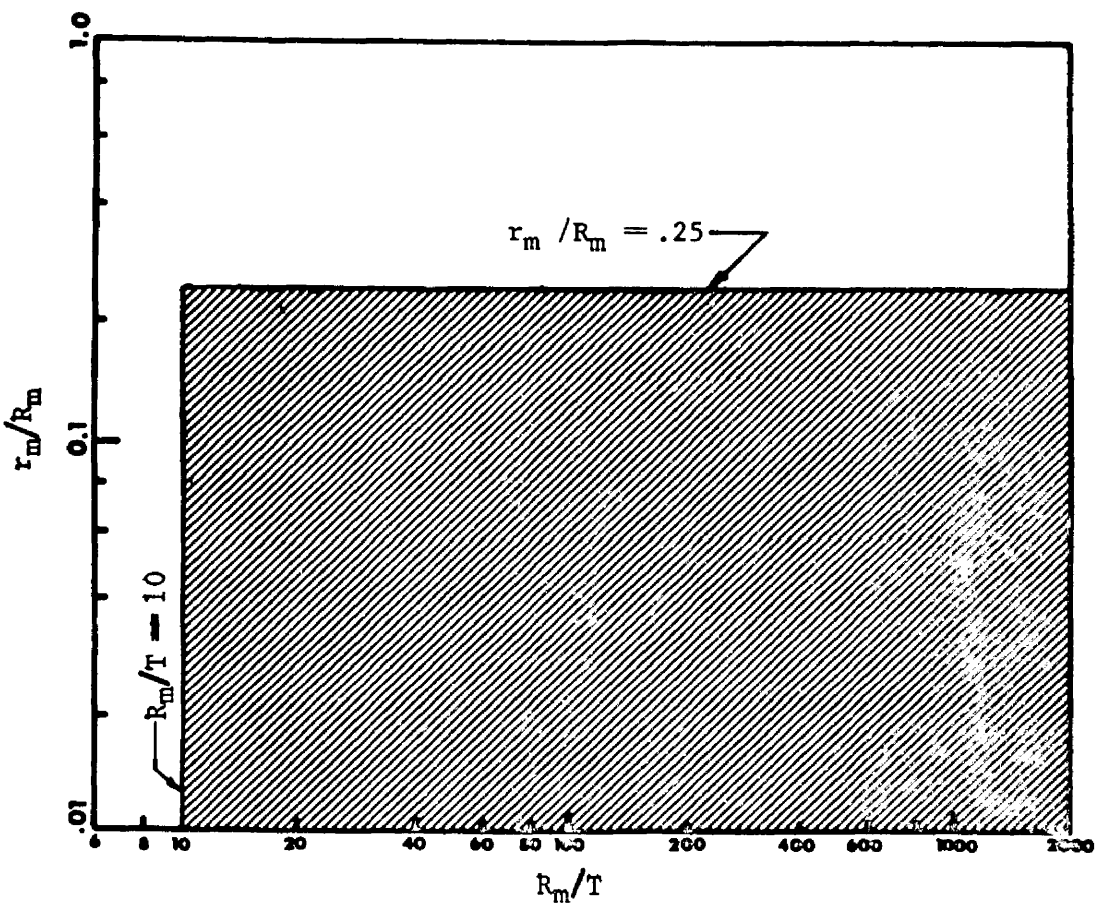

Fig. B7.2.2.1-4

B Location of Attachment with Respect to Boundary Conditions to Shell

The analysis is applicable when there are no stress perturbations caused by other loadings in the area influenced by the local loads. These perturbations can be caused by discontinuity, thermal loading, liquid level loading, change in section and material change. The area influenced by the local loading can be determined by an investigation of the stresses and deflections at points removed from the attachment.

C Shift in Maximum Stress Locations

Under certain conditions the stresses in the shell may be higher at points removed from the attachment-to-shell juncture than at the juncture. The following conditions should be carefully considered:

1. Stresses can be higher in the attachment than in the shell. This is most likely when the attachment is not reinforced, when reinforcement is placed on the shell and not on the attachment, and when very thin attachments are used.

2. For some load conditions certain stress resultants peak at points slightly removed from the attachment-to-shell juncture. The load conditions that cause this peaking are in most cases the same load cases that cause peaking for local loads on spherical shells. The extent of the peaking can be evaluated by an investigation of the stresses and deflections at points slightly removed from the attachment.

3. Comparison of analytical and experimental results [3] for membrane stresses shows that membrane stress resultants can be calculated at the point where stresses are desired. Comparison of analytical and experimental results [2, 3] for bending stress resultants at loaded attachments shows that the bending stress resultants must be calculated at the center of the attachment and

then shifted to the edge for the determination of stresses at the edge of the attachment. The determination of bending stresses at other points requires that the bending stress resultants be calculated at a distance $C_{1/2}$ or $C_{2/2}$ closer to the attachment.

D Stresses Caused by Shear Loads

An accurate stress distribution caused by a shear load (V_a) applied to a cylindrical shell is not available. The actual stress distribution consists of varying shear and membrane stresses around the rigid attachment. The method [2] presented here assumes that the shell resists the shear load by shear only. If this assumption appears unreasonable, it can be assumed that the shear load is resisted totally by membrane stresses or by some combination of membrane and shear stresses.

E Shell and Attachment Geometry

The analysis assumes that the cylindrical shell has simply supported end conditions or is of sufficient length that simply supported end conditions can be assumed.

The computer program requires that circular and elliptical attachments be converted to equivalent square and rectangular attachments, respectively. The equivalent attachment must have an area equal to the area of the actual attachment for a radially applied force. The equivalent attachment must have a moment of inertia about the bending axis equal to the moment of inertia about the bending axis of the actual attachment for bending loads. In both cases the aspect ratios (a/b and c_1/c_2) of the attachments (actual and equivalent, respectively) must be equal. If the attachment is welded, the weld size must be added to the attachment when determining equivalent attachments.

L/R_m is a secondary parameter and has little effect on the solution of $1.0 \leq L/R_m \leq 5.0$. The attachment coordinate system, defined by Figure B7.2.2.1-2, must be located at $x = L/2$.

F Shell Loading

The computer program accounts for pressure coupling (that is, the increase in shell stiffness caused by internal pressure). The internal pressure (q) must be positive or a positive differential. The stresses caused by the internal pressure must be calculated separately and superimposed upon the local loads stresses calculated by the method presented here.

The shell deflections must be small, approximately equal to the cylindrical shell thickness, for the analysis to be valid and to allow superposition of stresses.

B7.2.2.2 STRESSES

I GENERAL

Stress resultants and displacements caused by radial load (P) and overturning moment (M_a) are obtained from the computer program given in the Computer Handbook. Stresses caused by shear load (V_a) and twisting moment (M_T) are calculated directly from attachment geometry and loading.

The stress resultants (M_j and N_j) and displacements (u, v, and w) determined by the computer program are for a specific location. The location is specified by x and ϕ input values determined according to the coordinate system (x, ϕ, z) defined in Figure B7.2.2.1-2.

The computer program will calculate stress resultants and displacements for configurations (see Computer Utilization Handbook):

Case 1 - One Uniformly Distributed Radial Load

Case 2 - "n" Equally Spaced Uniformly Distributed Radial Loads

Case 3 - One Concentrated Radial Load

Case 4 - "n" Equally Spaced Concentrated Radial Loads

Case 5 - Longitudinal Overturning Moment

Case 6 - Circumferential Overturning Moment

The general equation for stresses in a shell at a rigid attachment juncture in terms of the stress resultants is of the form:

$$f_j = K_n (N_j/T) \pm K_b (6M_j/T^2)$$

The stress concentration parameters (K_n and K_b) are defined and can be evaluated from Paragraph B7.2.1.2, Sections I and IIB.

Figure B7.2.2.2-1 "Stress Calculation Sheet" can be used for the calculation of all stresses caused by an arbitrary local loading. The sheet automatically accounts for signs.

STRESS CALCULATION SHEET FOR STRESSES IN CYLINDRICAL SHELLS CAUSED BY LOCAL LOADS

APPLIED LOADS	SHELL GEOMETRY	CO-ORDINATES			PARAMETERS
			x	φ	
P = ______	T = ______	O =	______	______	K_n = ______
P = ______	R_m = ______	A =	______	______	K_b = ______
V_1 = ______	L = ______	B =	______	______	
V_2 = ______	c_1 = ______	C =	______	______	
M_T = ______	c_2 = ______	D =	______	______	
M_1 = ______	a = ______		______	______	
M_2 = ______	r_o = ______				

STRESS	LOAD	STRESS RESULT.	ADJUST. FACTOR	CALC. STRESS	STRESSES* A_i	STRESSES* A_o	STRESSES* C_i	STRESSES* C_o	STRESS RESULT.	ADJUST. FACTOR	CALC. STRESS	STRESSES* B_i	STRESSES* B_o	STRESSES* D_i	STRESSES* D_o
LONGITUDINAL STRESS (f_x)	P case I thru IV	$N_x(A)$ =**	$\frac{K_n}{T}$ =		+	+	+	+	$N_x(B)$ =**	$\frac{K_n}{T}$ =		+	+	+	+
		$M_x(O)$ =	$\frac{6K_b}{T^2}$ =		+	–	+	–	$M_x(O)$ =	$\frac{6K_b}{T^2}$ =		+	–	+	–
	M_1 case VI	$N_x(A)$ =	$\frac{K_n}{T}$ =						$N_x(B)$ =	$\frac{K_n}{T}$ =		+	+	–	–
		$M_x(A)$ =	$\frac{6K_b}{T^2}$ =						$M_x(B)$ =	$\frac{6K_b}{T^2}$ =		+	–	+	–
	M_2 case V	$N_x(A)$ =	$\frac{K_n}{T}$ =		–	–	+	+	$N_x(B)$ =	$\frac{K_n}{T}$ =					
		$M_x(A)$ =	$\frac{6K_b}{T^2}$ =		–	+	+	–	$M_x(B)$ =	$\frac{6K_b}{T^2}$ =					
	TOTAL LONGITUDINAL STRESSES (f_x)														
CIRCUMFERENTIAL STRESS (f_y)	P	$N_y(A)$ =**	$\frac{K_n}{T}$ =		+	+	+	+	$N_y(B)$ =**	$\frac{K_n}{T}$ =		+	+	+	+
		$M_y(O)$ =	$\frac{6K_b}{T^2}$ =		+	–	+	–	$M_y(O)$ =	$\frac{6K_b}{T^2}$		+	–	+	–
	M_1	$N_y(A)$ =	$\frac{K_n}{T}$ =						$N_y(B)$ =	$\frac{K_n}{T}$ =		+	+	–	–
		$M_y(A)$ =	$\frac{6K_b}{T^2}$ =						$M_y(B)$ =	$\frac{6K_b}{T^2}$ =		+	–	+	–
	M_2	$N_y(A)$ =	$\frac{K_n}{T}$ =		–	–	+	+	$N_y(B)$ =	$\frac{K_n}{T}$ =					
		$M_y(A)$ =	$\frac{6K_b}{T^2}$ =		–	+	+	–	$M_y(B)$ =	$\frac{6K_b}{T^2}$ =					
	TOTAL CIRCUMFERENTIAL STRESSES (f_y)														
SHEAR STRESS (f_{xy})	V_1	V_1 =	$\frac{1}{\pi r_o T}$ =									–	–	+	+
	V_2	V_2 =	$\frac{1}{\pi r_o T}$ =		–	–	+	+							
	M_T	M_T =	$\frac{1}{2\pi r_o^2 T}$ =		–	–	–	–				+	+	+	+
	TOTAL SHEAR STRESS (f_{xy})														
PRINCIPAL STRESSES	f max	$\frac{f_x + f_y}{2} + \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$ ***													
	f min	$\frac{f_x + f_y}{2} - \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$ ***													
	f_{xy} max	$\pm \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$													

* CHANGE SIGN OF CALCULATED STRESS WHERE NEGATIVE SIGNS ARE INDICATED.

** LETTER IN PARENTHESIS DESIGNATES THE POINT (A, B, OR O) AT WHICH THE STRESS RESULTANTS SHOULD BE COMPUTED BY THE COMPUTER PROGRAM. THE STRESSES FOR POINTS (A, B, C, AND D) CAN BE OBTAINED FROM THESE STRESS RESULTANTS USING THE SIGNS INDICATED.

*** CHANGE SIGN OF THE RADICAL IF $(f_x + f_y)$ IS NEGATIVE.

Fig. B7.2.2.2-1 Stress Calculation Sheet

B7.2.2.2 STRESSES

II STRESSES RESULTING FROM A RADIAL LOAD

Radial load configuration Cases I and II (Computer Utilization Handbook) cause membrane and bending stress components in both the longitudinal and circumferential directions.

A Longitudinal Stress (f_x)

Step 1. Determine the required load and geometric load input for the computer program.

Step 2. Determine the bending stress resultant (M_x) at point O and the normal stress resultant (N_x) at points A and B with the computer program. See Figure B7.2.2.1-2 for the location of points A, B and O.

Step 3. Using the criteria in Paragraph B7.2.1.2, Section I, obtain values for the stress concentration parameters (K_n and K_b).

Step 4. Using the bending stress resultant (M_x) at point O and the normal stress resultant (N_x) at point A as determined in Step 2, and the stress concentration parameters as determined in Step 3, determine the longitudinal stresses (f_x) at point A using the following equation:

$$f_x = K_n(N_x/T) \pm K_b(6M_x/T^2)$$

Proper consideration of the sign will give the values for the longitudinal stress at the inside and outside surfaces of the shell.

Step 5. Repeat Step 4, but use the normal stress resultant (N_x) as determined for point B and the bending stress resultant (M_x) as determined for point O to determine the longitudinal stresses (f_x) at point B.

B Circumferential Stresses (f_y)

The circumferential stresses (f_y) can be determined by following the five steps outlined in Paragraph A, above, except for determining M_y and N_y instead of M_x and N_x in Step 2 and using the following stress equation in Step 4.

$$f_y = K_n(N_y/T) \pm K_b(6M_y/T^2)$$

C Concentrated Load Stresses

Points A, B, C and D in Figure B7.2.2.1-2 do not exist for Load Cases III and IV. Longitudinal and circumferential membrane and bending stress caused by concentrated loads (Cases III and IV) is determined from stress resultants calculated at point O. The stresses are calculated using Paragraph A, above, after applying proper modifications to the equations.

B7.2.2.2 STRESSES

III STRESSES RESULTING FROM AN OVERTURNING MOMENT

Overturning moment load configurations, Cases V and VI (Computer Utilization Handbook), will cause membrane and bending stress components in both the longitudinal and circumferential directions.

A Longitudinal Stress (f_x)

Step 1. Determine load and geometric input for the computer program.

Step 2. Determine the bending stress resultant (M_x) and normal stress resultant (N_x) at points A for Load Case V and B for Load Case VI with the computer program.

Step 3. Using the criteria in Paragraph B7.2.1.2, Section I, obtain the values for the stress concentration parameters (K_n and K_b).

Step 4. Using the bending stress resultant (M_x) and the normal stress resultant (N_x) at point A as determined in Step 2 and the stress concentration parameters as determined in Step 3, determine the longitudinal stress (f_x) at point A using the following equation:

$$f_x = K_n(N_x/T) \pm K_b(6M_x/T^2)$$

Proper consideration of the sign will give the values for longitudinal stresses at the inside and outside surfaces of the shell.

Step 5. Repeat Step 4, but use stress resultants as determined for point B to determine the longitudinal stresses at point B.

B Circumferential Stress (f_y)

The circumferential stresses (f_y) can be determined by following the five steps outlined in Paragraph A, above, except for determining M_y and N_y instead of M_x and N_x in Step 2, and using the following stress equation in Step 4:

$$f_y = K_n (N_y/T) \pm K_b (6M_y/T^2)$$

B7.2.2.2 STRESSES

IV STRESSES RESULTING FROM A SHEAR LOAD

A shear load (V_a) will cause a shear stress (f_{xy}) in the shell at the attachment-to-shell juncture. The shear stress is determined as follows:

A Round Attachment

$$f_{xy} = \frac{V_a}{\pi r_0 T} \sin \theta \qquad \text{for } V_a = V_1$$

$$f_{xy} = \frac{V_a}{\pi r_0 T} \cos \theta \qquad \text{for } V_a = V_2$$

B Rectangular Attachment

$$f_{xy} = \frac{V_a}{4c_1 T} \qquad \text{for } V_a = V_1$$

$$f_{xy} = \frac{V_a}{4c_2 T} \qquad \text{for } V_a = V_2$$

B7.2.2.2 STRESSES

V STRESSES RESULTING FROM A TWISTING MOMENT

A Round Attachment

A twisting moment (M_T) applied to a round attachment will cause a shear stress (f_{xy}) in the shell at the attachment-to-shell juncture. The shear stress is pure shear and is constant around the juncture. The shear stress is determined as follows:

$$f_{xy} = M_T/2\pi r_0 T$$

B Square Attachment

A twisting moment applied to a square attachment will cause a complex stress field in the shell. No acceptable methods for analyzing the loading are available.

B7.2.2.3 STRESS RESULTING FROM ARBITRARY LOADING

I CALCULATION OF STRESSES

Most loadings that induce loads on cylindrical shells are of an arbitrary nature. Stresses are determined by the following procedure:

Step 1. Resolve the arbitrary applied load (forces and/or moments) into axial force, shear forces, overturning moments and twisting moment components. See Paragraph B7.2.1.6 Example Problem. The positive directions of the components and the point of application of the load components (intersection of centerline of attachment with attachment-shell interface) are indicated in Figure B7.2.2.1-1.

Step 2. Evaluate inside and outside stresses at desired points (such as A, B, C and D) around the attachment for each component of the arbitrary applied loading by the methods in Paragraph B7.2.2.2.

Step 3. Obtain the stresses for the arbitrary loading by combining the longitudinal, circumferential and shear stresses evaluated by Step 2 for each of the points selected on the inside and outside of the shell. Proper consideration of signs is necessary.

B7.2.2.3 STRESSES RESULTING FROM ARBITRARY LOADING

II LOCATION AND MAGNITUDE OF MAXIMUM STRESS

The location and magnitude of the maximum stresses caused by an arbitrary load require a consideration of the following:

A. The determination of principal stresses (f_{max}, f_{min}, $f_{xy} = 0$, or $f_{xy} = max$) for the determined stresses (f_x, f_y, and f_{xy}) at a specific point.

B. The proper selection of points for determining the stresses.

B7.2.2.4 DISPLACEMENTS

Shell displacements caused by radial load configurations and overturning moments are obtained from the computer program described in the Computer Handbook. Shell displacements caused by twisting moment and shear loads are not determined.

Comparison of experimental and theoretical deflections indicate that deflections are sensitive to the detailed conditions of the attachment. In general, however, the experimental and theoretical values are of the same order of magnitude.

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