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16. ABSTRACT

This document (Volumes I, II, and III) presents a compilation of industry-wide methods in aerospace strength analysis that can be carried out by hand, that are general enough in scope to cover most structures encountered, and that are sophisticated enough to give accurate estimates of the actual strength expected. It provides analysis techniques for the elastic and inelastic stress ranges. It serves not only as a catalog of methods not usually available, but also as a reference source for the background of the methods themselves.

An overview of the manual is as follows: Section A is a general introduction of methods used and includes sections on loads, combined stresses, and interaction curves; Section B is devoted to methods of strength analysis; Section C is devoted to the topic of structural stability; Section D is on thermal stresses; Section E is on fatigue and fracture mechanics; Section F is on composites; Section G is on rotating machinery; and Section H is on statistics.

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SECTION B7

THIN SHELLS

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B7.0 Thin Shells

Basic relationships governing the behavior of shells whose thicknesses are small relative to their surface dimensions and to their principal radii of curvature are summarized in this section. This thinness admits various approximations to the three-dimensional stress state. The degree of approximation best suited for a particular analysis depends on the shell shape, the type of loading, and the material of which the shell is made. Consequently, there exists a variety of approximate thin-shell theories.

The various thin shell theories to be used in subsequent analyses are discussed below. The purpose is to familiarize the analyst with the foundations upon which commonly employed shell equations are based.

B7.0.1 Thin Shell Theories

Theories of thin shells may be broadly classified according to the fundamental theories which they approximate:

I The Theory of Linear (classical) Elasticity

II Nonlinear Elasticity

III Inelasticity

The most common shell theories are those based on linear elasticity concepts. These theories adequately predict stresses and deformations for shells exhibiting small elastic deflections; they are also adaptable to some buckling problems.

The Nonlinear Theory of Elasticity forms the basis for finite and large deflection theories of shells. These theories are often required when dealing with shallow shells, buckling problems, and highly elastic membranes. The nonlinear shell equations are considerably more difficult to solve and therefore are more limited in use.

Shells in the inelastic range will not be discussed in this section.

B7.0.2 Thin Shell Theories Based on Linear Elasticity

The classical three-dimensional equations of Linear Elasticity are based upon the following assumptions:

1. Displacement gradients are small; i.e.,

$$\frac{u_i}{x_j} \ll 1$$

where u_i = generalized displacement, i = 1, 2, 3

x_j = generalized coordinate, j = 1, 2, 3

2. Products of displacement gradients are therefore negligible compared to the gradients themselves. By this assumption, strains and rotations are necessarily small and they become linear functions of the displacement gradients, i.e.,

$$\epsilon_1 = \frac{\partial u_1}{\partial x_1}; \quad \epsilon_2 = \frac{\partial u_2}{\partial x_2}; \quad \gamma_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1};$$

etc.

3. It is further assumed that the Generalized Hooke's Law holds, an assumption which is naturally compatible with the small strain condition. Hooke's Law, in its general form, states that the six components of stress at any point are linear functions of the six components of strain at that point.

When dealing with thin plates or shells, the stresses in planes parallel to the surface are of prime importance, the normal stresses being of little practical significance. Hence, a complete three-dimensional solution is generally not warranted. Sufficiently accurate analyses of thin plates and shells can be performed using simplified versions of the general Linear Elasticity equations.

The selection of the proper form of these approximations has been the subject of considerable controversy among the many investigators in the field. As a result, there is in existence a large number of general and specialized thin shell theories, developed within the framework of linear elasticity. The most commonly encountered theories will be discussed in the subsequent sections and classified according to the assumptions upon which they are based.

The various linear shell theories will be classified into five basic categories:

1. First-Order Approximation Shell Theory

2. Second-Order Approximation Shell Theory

3. Shear Deformation Shell Theory

4. Specialized Theories for Shells of Revolution

5. Membrane Shell Theory

In the case of thin shells, the simplified bending theories of shells are (in general) based on Love's first-approximation and second-approximation shell theories. Although some theories do not adhere strictly to Love's two original approximations, they can be considered as modifications thereof and will be categorized as either a first or second approximation.

Although the Shear Deformation and Specialized Shell theories presented are based on Love's first-approximation, they are classified separately because of their particular physical significance.

Linear membrane theory is the limiting case corresponding to a zero-order approximation, or momentless state.

B7.0.2.1 First-Order Approximation Shell Theory

Love was the first investigator to present a successful approximate shell theory based on linear elasticity. To simplify the strain-displacement relationships and, consequently, the stress-strain relations, Love introduced the following assumptions, known as first approximations and commonly termed the Kirchhoff-Love hypothesis:

1. The shell thickness, t, is negligibly small in comparison with the least radius of curvature, R_{min}, of the middle surface; i. e., $\frac{t}{R_{min}} << 1$ (therefore, terms $\frac{z}{R} << 1$).

2. Linear elements normal to the unstrained middle surface remain straight during deformation, and their extensions are negligible.

3. Normals to the undeformed middle surface remain normal to the deformed middle surface.

4. The component of stress normal to the middle surface is small compared with other components of stress, and may be neglected in the stress-strain relationships.

5. Strains and displacements are small so that quantities containing second-and higher-order terms are neglected in comparison with first-order terms in the strain equations.

The last assumption is consistent with the formulation of the classical theory of linear elasticity. The other assumptions are used to simplify the elasticity relations.

By the thickness condition, assumption (1) above, the ratios $\frac{z}{r_1}$ and $\frac{z}{r_2}$ are negligible relative to unity. From this condition, the ten stress resultants that act on an infinitesimal element (N_ϕ, N_θ, Q_ϕ, Q_θ, $Q_{\theta\phi}$, $Q_{\phi\theta}$, M_ϕ, M_θ, $M_{\phi\theta}$, and $M_{\theta\phi}$) reduces to eight, since $Q_{\phi\theta} = Q_{\theta\phi}$ and $M_{\phi\theta} = M_{\theta\phi}$.

Assumption (2) of Love's first approximation is analogous to Navier's hypothesis in elementary beam theory, i. e., plane sections remain plane during bending.

The strain equations are further simplified through assumption (3), by which transverse shear deformations are neglected. As a consequence, normals to the middle plane not only remain straight but remain normal and have the same rotation as the middle surface. The degree of error introduced by this assumption naturally depends on the magnitude of transverse shearing forces; in local areas around a shell edge such shear deformations may be comparable to bending and axial deformations, and cannot be ignored. In general, however, shells loaded by continuously distributed surface forces, and having flexibly supported edges, can be assumed to have negligible transverse shear deformations.

By the fourth assumption, forces applied to the surface of the shell are stated to be so distributed that directly imposed stresses are small. Furthermore, direct normal stresses through the thickness, f_z, are taken to be insignificant due to the large radius-to-thickness ratios of the shell.

Practically speaking, the solution of the simultaneous differential equations of Love's first order approximation theory is possible only in rare cases, or with additional approximations. In the case of a loaded structure, the general solution of the nonhomogeneous differential equations consists of a particular solution of the nonhomogeneous differential equations and the general solution of the homogeneous differential equations. In the case of an unloaded structure the solution consists of only the general solution of the homogeneous differential equations.

The nonhomogeneous solution of Love's equations, to a first approximation, equals the solution of the corresponding extensional (pure membrane) problem. The homogeneous solution is a self-equilibrating system of stress resultants which satisfy compatibility conditions at the edges of the shell (edge effect) and in other regions of discontinuity.

Thus, there are two extreme cases possible within the first approximation: (1) the inextensional or pure bending case in which middle plane strains are neglected compared with flexural strains, and (2) the extensional or membrane case in which only middle plane strains are considered. The general or mixed case lies between these two extremes.

B7.0.2.2 Second-Order Approximation Shell Theory

In Love's second approximation, restrictions on the t/r ratios are relaxed to such an extent that normal stresses induced by flexure and corresponding normal displacements are no longer negligible. By considering the second-order effects of such normal displacements, the strain components parallel to the middle surface become nonlinear functions of middle-plane curvature changes.

Assumptions (2) and (3) of the first-order theory are retained in the second approximation. Thus, displacements are said to vary linearly across the thickness of the shell, whereas, strains are nonlinearly distributed.

It is characteristic of second approximation theories that strains and constitutive relations contain second-order terms in the thickness coordinate, z.

The theory is applicable for small deflections of highly curved shells subjected to predominantly flexural strains.

B7.0.2.3 Shear Deformation Shell Theories

In the development of the first-and second-order shell theories the effects of transverse shear deformation were neglected. This neglect resulted because of the geometrical assumptions that normals remain normal. It is possible that for some loads or shell configurations, the transverse shear strains can no longer be neglected and, therefore, these effects must be included in the theory.

When the effects of shear deformation are included, the shear strains no longer vanish and, as a result, the rotation expressions are no longer determinate.

Since the shear forces are now related to deformations, they can be eliminated from equilibrium equations. Thus, five boundary conditions are necessary at each boundary (Reference 1).

B7.0.2.4 Specialized Theories for Shells of Revolution

The bending shell theories previously discussed can be simplified considerably for specialized conditions of geometry and loading. Some of the simplified shell theories resulting from consideration of shells of revolution of specific geometry will be presented in this section. These theories are based on first-order approximation; however, for purposes of illustration they are classified separately. In this section, the simplified shell theories are presented for shells of particular interest. Included are the Reissner-Meissner theories, Geckeler's approximations, shallow-shell theory, Donnell's theory, and others.

I. General Shells of Revolution Axisymmetrically Loaded

For the case of axisymmetrical deformation, the displacement in the Θ direction ($\bar{v}$) is zero, and all derivatives of displacement components with respect to Θ are also zero. From symmetry, the resultant forces $Q_{\phi\theta}$, Q_{θ}, and $M_{\phi\theta}$ vanish. Two second-order ordinary differential equations in the two unknown displacement components $\bar{u}$ and $\bar{w}$ can be obtained. Rather than obtain equations in this manner, however, a transformation of dependent variables can be performed leading to a more manageable pair of equations which, for shells of constant meridional curvature and constant thickness, combine into a single fourth-order equation solvable in terms of a hypergeometric series. Historically, such a transformation of variables was first introduced by H. Reissner (1913) for spherical shells and then generalized to all shells of constant thickness and constant meridional curvature by E. Meissner (1914). Meissner next showed (1915) that the equations for a general shell of revolution are also transformable to Reissner-Meissner type equations provided the thickness t and the radius r_1 both vary so as to satisfy a certain relationship for all values of ϕ, (the "Meissner condition").

Reissner-Meissner type equations are the most convenient and widely employed forms of the first-approximation theory for axisymmetrically loaded shells of revolution. It is seen that they follow exactly from the relations of Love's first approximation when the meridional curvature and thickness are constant, as they are for cylindrical, conical, spherical, and toroidal shells of uniform thickness. Furthermore, they follow directly from Love's equations in the more general case provided special restraints on the variation of thickness and geometry are satisfied.

Using a more recent version of the Reissner-Meissner equations (Reference 2), toroidal shells of constant thickness were investigated by Clark (Reference 3) and ellipsoidal shells of constant thickness by Naghdi and DeSilva (Reference 4). In the latter case, the Meissner type condition, which would require the radius r_1 to be constant, is obviously not satisfied. It is shown, however, that assuming the Meissner condition to be satisfied yields a justifiable approximation for ellipsoidal shells.

II. Spherical Shells

For axisymmetrically loaded spherical shells of constant thickness, two simplified versions of the Reissner-Meissner equations are of engineering interest, namely Geckeler's approximation for nonshallow spherical shells and the Esslinger approximation for shallow shells.

For axisymmetrically loaded spherical shells of constant thickness, the fourth-order differential equation is:

$$\frac{d^4Q_\phi}{d_\phi{}^4} + A_3 \frac{d^3Q_\phi}{d_\phi{}^4} + A_2 \frac{d^2Q_\phi}{d_\phi{}^2} + A_1 \frac{dQ_\phi}{d_\phi} + A_oQ_\phi + 4\lambda^4Q_o = 0$$

where

$$A_o = 1-3\csc^4\phi - \mu^2$$

$$A_1 = \cot\phi(2+3\csc^2\phi)$$

$$A_2 = 1-3\csc^2\phi$$

$$A_3 = 2\cot\phi$$

and

$$\lambda^4 = 3(1-\mu^2)\frac{R^2}{t^2}$$

In the Geckeler approximation all terms except the first and last in the equation above are neglected, leaving:

$$\frac{d^4 Q_\phi}{d\phi^4} + 4 \lambda^4 Q_o = 0$$

Geckeler's equation is seen to be of the same form as the equation for the beam on an elastic foundation.

This approximation is valid for large values of λ and high angles ϕ; that is, for thin, non-shallow spherical shells. The approximation is particularly good in the vicinity of $\phi = 90^o$, however, it is considered to be sufficiently accurate for angles as small as $\phi = 20^o$.

For small angles ϕ, the Reissner-Meissner equations can be approximated by making the usual low angle assumption that sin $\phi \approx \phi$ and cos $\phi \approx 1$, a simplification considered in detail by Esslinger. The solution of these equations is in terms of derivatives of Schleicher functions.

Another approximation for non-shallow shells is based on the transformation:

$$\bar{Q}_\phi = Q_\phi \sqrt{\sin \phi}$$

This involves a slightly more accurate approximation than Geckeler's, and was introduced by O. Blumenthal. Complete solutions were given by Hetényi (Reference 5).

III. Circular Cylindrical Shells

For the case of circular cylindrical shells arbitrarily loaded, two first approximate theories are of prime importance: Love's first-approximation theory, and its simplified version due to Donnell.

Donnell simplified the strain displacement relations by ignoring the influence of the original shell curvature on the deformations due to bending and twisting moment. By this approximation the relations between moments and change in curvature and twist become the same as for flat plates.

Donnell's equations are specially applicable to shell stability problems (Reference 6 and section on shell stability), however, in their homogeneous form they have been widely used for problems of circular cylinders under line loads, concentrated loads, and arbitrary edge loads. A review of such solutions is presented in Reference 7.

IV. Second-Order Approximation Theories for Shells of Revolution

The second-order approximation theory of Flügge (Reference 8) and Byrne (Reference 9) retain the z/r terms with respect to unity in the stress resultant equations and in the strain-displacement relations. Flügge - Byrne type equations for a general shell are discussed by Kempner (Reference 10) who obtains them as a special case of a unified thin-shell theory. Applications of this second approximation theory have generally been restricted to circular cylindrical shapes, for which case solutions are obtained in References 9 and 11. In the latter reference the Flügge - Byrne type equations are considered as standards with which simplified first-approximation theories are compared.

Second-approximation equations are derived by Vlasov directly from the general three-dimensional Linear Elasticity equations for a thick shell (Reference 12). An excellent discussion of the assumptions made by Vlasov is given by Novozhilov (Reference 13).

V. Membrane Theory of Shells

The shell theories studied in the previous sections are generally referred to as "bending" theories of shells because this development includes the consideration of the flexural behavior of shells. If, in the study of equilibrium of a shell all moment expressions are neglected, the resulting theory is the so-called "membrane" theory of shells. Membrane analysis of shells is presented in Section B7. 1.

B7.0.3 Nonlinear Shell Theory

The small-deflection field theories discussed in the previous sections were formulated from the classical linear theory of elasticity. It is known that these operations, which are based on Hooke's Law and the omission of nonlinear terms both in the equations for strain components and the equilibrium equations, have a unique solution in every case. In other words, linear shell theory determines a unique position of equilibrium for every shell with prescribed load and constraints.

In reality, however, the solution of a physical shell problem is not always unique. A shell under identical conditions of loading and constraints may have several possible positions of equilibrium. The incorrect inference to which linear shell theory leads can be explained by the approximations introduced in the development of the shell equations. In this development, rotations were neglected in the expressions for strains and equilibrium in order that the equations could be linearized. It is essential in the investigation of the multiple-equilibrium states of a shell to include these rotation terms.

A theory of shells that is free of this hypothesis can be thought of as being "geometrically nonlinear" and requires formulation on the basis of the nonlinear elasticity theory. Additionally, the shell may be "physically nonlinear" with respect to the stress-strain relations. This latter type of nonlinearity forms the basis of inelastic shell theory and will not be discussed here.

The development of nonlinear shell theory is based on a general mathematical approach described by Novozhilov (Reference 14) for problems of nonlinear elasticity. Starting with the general strain-displacement relations, approximate nonlinear strain-displacement relations and equilibrium equations are derived by the introduction of appropriate simplifying assumptions. The equilibrium equations are obtained upon application of the principle of stationary potential energy.

Theories based on nonlinear elasticity are required in analyzing the so-called "large" deformation of shells. "Large" or finite deflection shell theories form the basis for the investigation of the stability of shells. In the case of stability, the effects of deformation on equilibrium cannot be ignored. The stability of shells will be considered in greater detail in Section C3.0.

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