SECTION B5

FRAMES

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B 5.0.0 FRAMES

This section deals with specialized methods of analyzing statically indeterminate structures. The procedures of analyzing rigid bents and circular rings are given in detail in Section B 5.1.0 and B 5.2.0 respectively.

A sample problem to illustrate the methods and procedures is given in each section.

B 5.1.0 Analysis of Statically Indeterminate Frames by the Method of Moment-distribution.

Moment-distribution is a convenient method of reducing statically indeterminate structures to a problem in statics. Moment-distribution does not involve the solution of simultaneous equations, but consists of a series of converging cycles that may be terminated at the degree of precision required by the problem.

B 5.1.1 Discussion of the Method of Moment-distribution.

The method of moment-distribution requires a knowledge of moment-area theorems and slope-deflection equations. The five basic factors involved in the method of moment-distribution are: fixed-end moments, stiffness factors, distribution factors, distributed moments, and carry-over moments.

Only structures comprised of prismatic members with no joint translation are considered in this article. All members are assumed to be elastic.

The fixed-end moments are obtained through the use of a general "end-moment" equation derived in the development of the slope-deflection method. (See Fig. B 5.1.1-1)

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - 3\psi_{AB}\right) + \frac{2}{L^2}\left[\left(\Omega o\right)_A - 2\left(\Omega o\right)_B\right] \quad \text{.........} \quad (1)$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - 3\psi_{AB}\right) + \frac{2}{L^2}\left[2\left(\Omega o\right)_A - \left(\Omega o\right)_B\right] \quad \text{.........} \quad (2)$$

where

M_{AB} is the moment acting on the "A" end of member AB.
M_{BA} is the moment acting on the "B" end of member AB.
E is the modulus of elasticity.
I is the moment of inertia.
L is the length of member AB.
θ is the rotation of the tangent to the elastic curve at the end of a member and is positive for clockwise rotation.
ψ is the rotation of the chord joining the ends of the elastic curve referred to the original direction of the member and is positive for clockwise rotation.
$(\Omega o)_A$ is the static moment about a vertical axis through "A" of the area under the M_o portion of the bending-moment diagram shown on Fig. B 5.1.1-1.
$(\Omega o)_B$ is the static moment about an axis through "B".

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

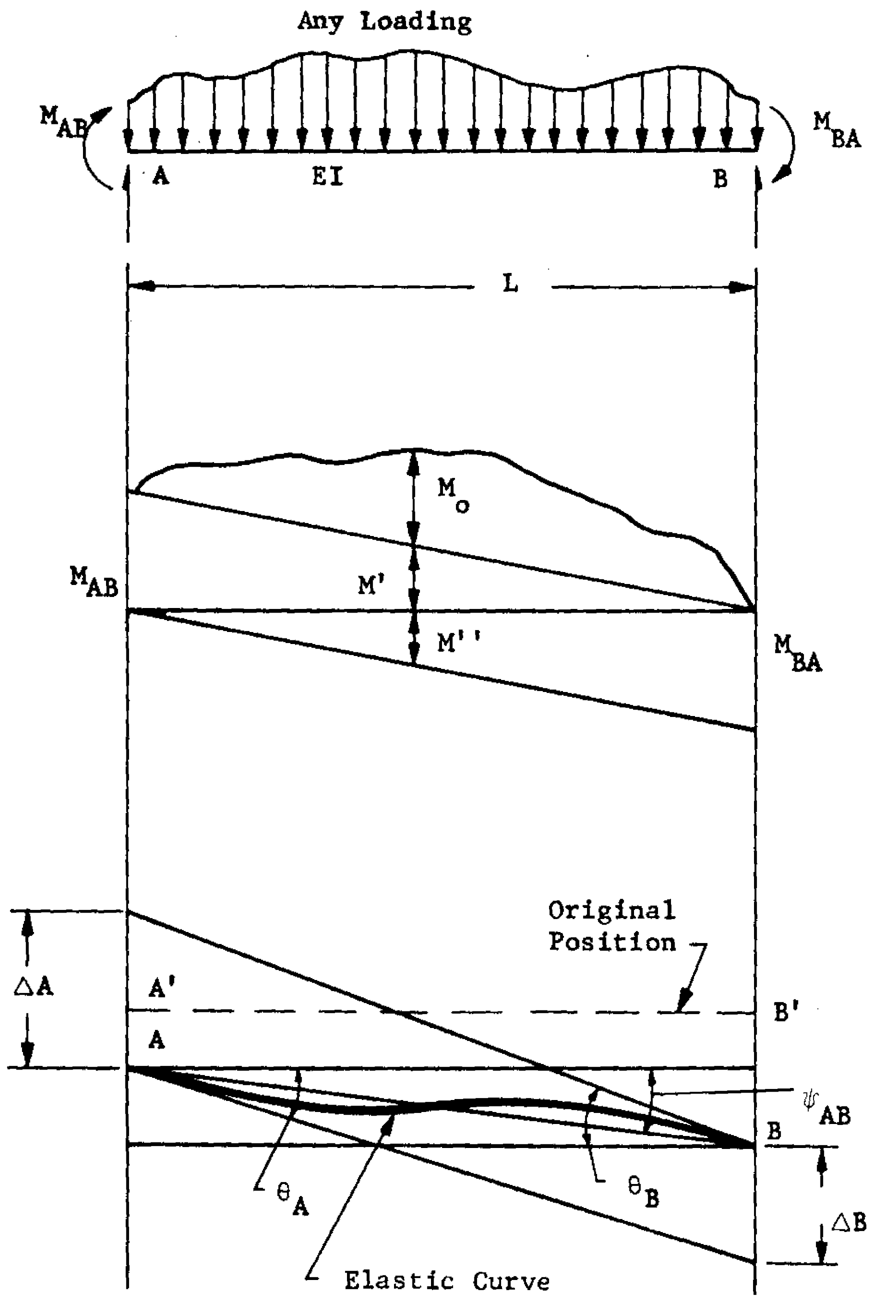


Fig. B 5.1.1-1

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

If θ_A, θ_B and ψ_{AB} are all equal to zero, then both ends of the member are completely fixed against rotation or translation and the member is called a fixed-end beam. The last terms of Eqs. (1) and (2) are therefore equal to the so-called "fixed end moments". Denoting fixed end moments as FEM and setting θ_A, θ_B and ψ_{AB} equal to zero.

$$FEM_{AB} = \frac{2}{L^2}\left[(\Omega o)_A - 2\,(\Omega o)_B \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$FEM_{BA} = \frac{2}{L^2}\left[2(\Omega o)_A - (\Omega o)_B \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Fixed end moments for various type of loading are given in Table B 5.1.1-1

Equations (3) and (4) are summarized by one general equation by calling the near end of a member "N" and the far end "F". Also let

$$K_{NF} = \text{stiffness factor for member NF} = \frac{I_{NF}}{L_{NF}} \quad \ldots\ldots\ldots\ldots (5)$$

then the fundamental slope deflection equation is

$$M_{NF} = 2EK_{NF}\,(2\theta_N + \theta_F - 3\psi_{NF}) + FEM_{NF} \quad \ldots\ldots\ldots\ldots (6)$$

The conditions to be met at a joint are: (1) that the angle of rotation be the same for the ends of all members that are rigidly connected at that joint and (2) that the algebraic sum of all moments is zero. The method of moment-distribution renders to zero by iteration any unbalance in moment at a joint to satisfy the latter condition.

To distribute the unbalanced moment mentioned above, a distribution factor DF is required. This factor represents the relative portion of the unbalanced moment which is reacted by a member and for any bar bm, DF_{bm} is given by

$$DF_{bm} = \frac{K_{bm}}{\sum_b K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where the summation is meant to include all members meeting at joint b.

The distributed moment in any bar bm is then

$$M_{bm} = -DF_{bm}M \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Equation (8) may be interpreted as follows:

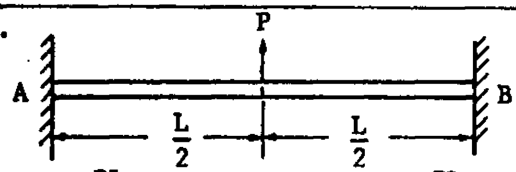
"The distributed moment developed at the 'b' end of member 'bm' as joint 'b' is unlocked and allowed to rotate under an unbalanced moment 'M' is equal to the distribution factor DF_{bm} times the unbalan moment 'M' with the sign reversed."

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

Table B 5.1.1-1 Fixed-end Moments for Beams

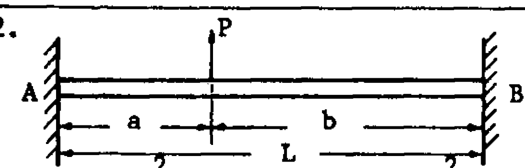
Notation: P = load (lb); w = unit load (lb per linear in.).
M = bending moment (in.-lb) positive when clockwise.

1.



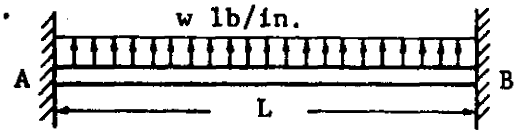
$M_A = \frac{PL}{8}$ $\qquad M_B = -\frac{PL}{8}$

2.



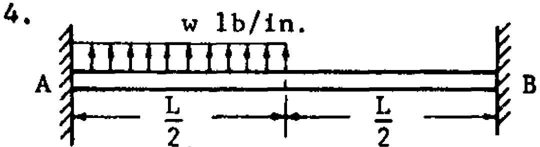
$M_A = \frac{Pab^2}{L^2}$ $\qquad M_B = -\frac{Pa^2b}{L^2}$

3.



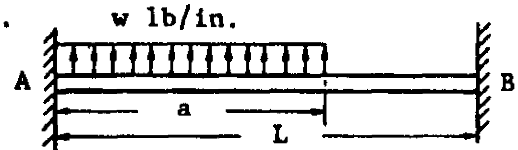
$M_A = \frac{wL^2}{12}$ $\qquad M_B = -\frac{wL^2}{12}$

4.



$M_A = \frac{11wL^2}{192}$ $\qquad M_B = -\frac{5wL^2}{192}$

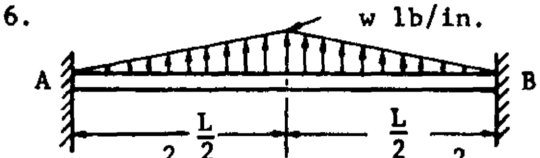
5.



$M_A = \frac{wa^2}{12L^2}(6L^2-8aL+3a^2)$

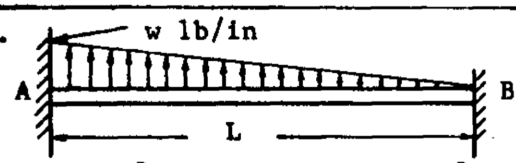
$M_B = -\frac{wa^2}{12L^2}(4aL-3a^2)$

6.



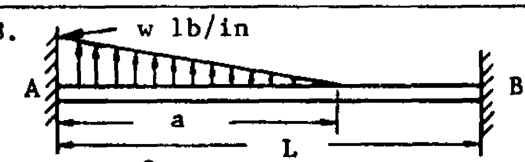
$M_A = \frac{5wL^2}{96}$ $\qquad M_B = -\frac{5wL^2}{96}$

7.



$M_A = \frac{wL^2}{20}$ $\qquad M_B = -\frac{wL^2}{30}$

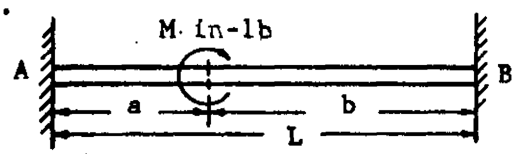
8.



$M_A = \frac{wa^2}{60L^2}(10L^2-10aL+3a^2)$

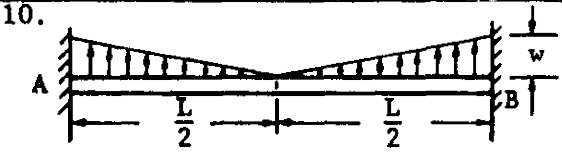
$M_B = -\frac{wa^3}{60L^2}(5L-3a)$

9.



$M_A = \frac{Mb}{L}(3\frac{a}{L} - 1)$, $M_B = -\frac{Ma}{L}(3\frac{b}{L} - 1)$

10.

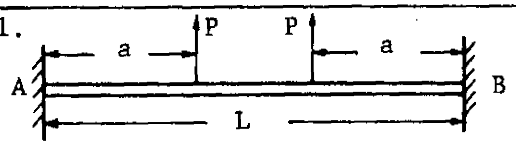


$M_A = \frac{wL^2}{32}$ $\qquad M_B = -\frac{wL^2}{32}$

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

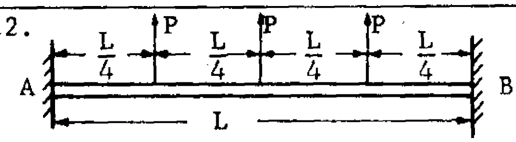
Table B 5.1.1-1 Fixed-end Moments for Beams (Cont'd)

11.



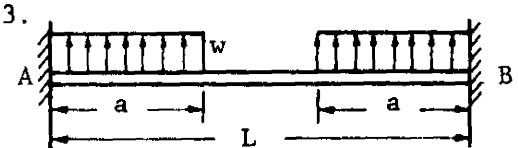
$M_A = Pa(1 - \frac{a}{L})$ $\qquad M_B = - M_A$

12.



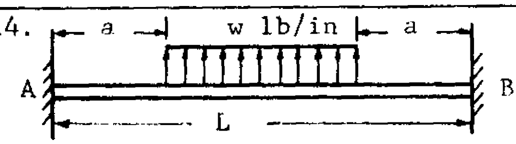
$M_A = \frac{15PL}{48}$ $\qquad M_B = - M_A$

13.



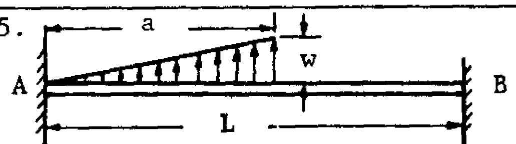
$M_A = \frac{wa^2}{6L}(3L-2a)$ $\qquad M_B = - M_A$

14.



$M_A = \frac{w}{12L}(L^3-a^2L+4a^3)$ $\qquad M_B = - M_A$

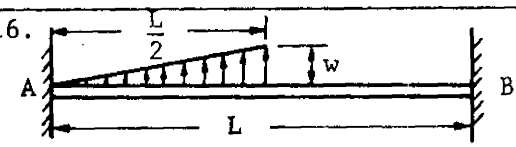
15.



$M_A = \frac{wa^2}{30}(10 - 15\frac{a}{L} + 6\frac{a^2}{L^2})$

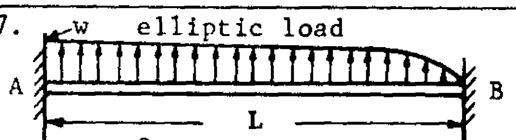
$M_B = \frac{wa^3}{20L^2}(5L-4a)$

16.



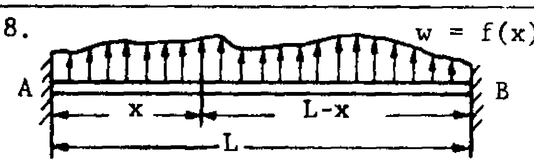
$M_A = \frac{wL^2}{30}$ $\qquad M_B = - \frac{3wL^2}{160}$

17.



$M_A = \frac{wL^2}{13.52}$ $\qquad M_B = - \frac{wL^2}{15.86}$

18.



$$M_A = \frac{1}{L^2}\int_0^L x(L-x)^2 f(x)dx$$

$$M_B = \frac{-1}{L^2}\int_0^L x^2(L-x)f(x)dx$$

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

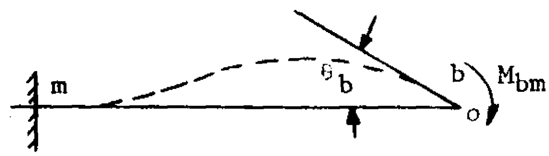


Fig. B 5.1.1-2

The "carry-over" moment is obtained by applying Eq. (6) considering $\theta_m = \psi_{bm} = o$ (see Fig. B 5.1.1-2). The carry-over moment is equal to one-half of its corresponding distributed moment and has the same sign.

$M_{bm} = 4EK_{bm}\theta_b$ and $M_{mb} = 2EK_{bm}\theta_b$

Hence,

$$M_{mb} = \frac{1}{2} M_{bm} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

The sign convention adopted for this work deviates from the usual convention used in elementary beam analysis as found in Sec. B 4.1.1 of this manual. The positive sense for moments has been adopted from the convention used in slope-deflection equations; namely, moments acting clockwise on the ends of a member are positive. (See Fig. B 5.1.1-3)

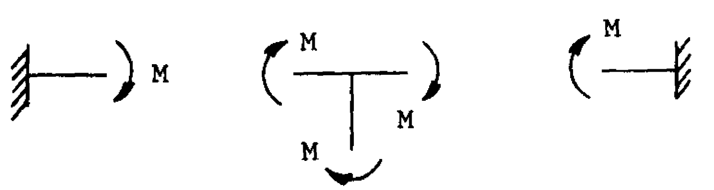


Fig. B 5.1.1-3 Positive sense for bending moments

The following procedure is established for the process of moment-distribution analysis:

1. Compute the stiffness factor for each member and record as shown in example problem one.

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

2. Compute the distribution factor for each member at each joint and record as shown in example problem one.

3. Compute the fixed-end moments for each loaded span and record with proper signs as shown in example problem one.

4. Balance the moments at a joint by multiplying the unbalanced moment by the distributor factor, changing sign, and recording the balancing moment below the fixed-end moment. The unbalanced moment is the algebraic sum of the fixed-end moments of a joint.

5. Draw a horizontal line below the balancing moment. The algebraic sum of all moments at any joint above the horizontal line must be zero.

6. Record the carry-over moment at the opposite ends of the member. Carry-over moments have the same sign as the corresponding balancing moments and are one-half their magnitude.

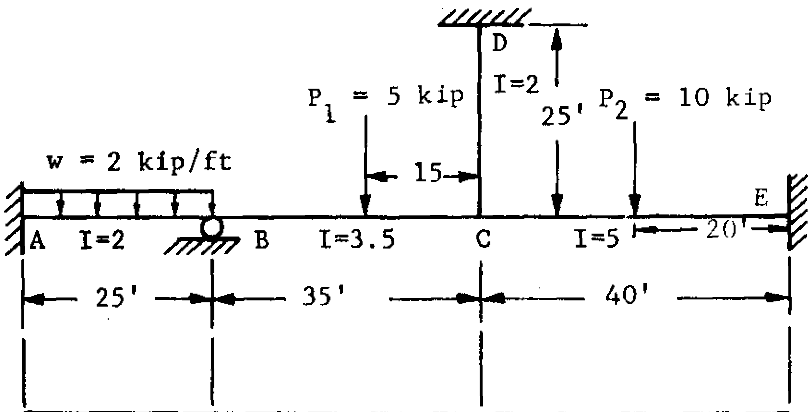
7. Move to a new joint and repeat the process for the balance and carry-over of moments for as many cycles as desired to meet the accuracy required by the problem. The unbalanced moment for each cycle will be the algebraic sum of the moments at the joint recorded below the last horizontal line.

8. Obtain the final moment at the end of each member as the algebraic sum of all moments tabulated at this point. The total of the final moments for all members at any joint must be zero.

9. Reactions, vertical shear, and bending moments of the member may be found through statics by utilizing the above mentioned final moments (step 8).

B 5.1.2 Sample Problem.

PROBLEM #1. Compute the end moments and draw the shear and bending-moment diagrams for this frame.



	AB	BA	BC	DC	CB	CE	CD	EC
K		.08	.1		.1	.125	.08	
DF		.44	.56		.33	.41	.26	
FEM	104	-104	+18	0	-25	+50	0	-50
		+38	+48		-8	-10	-7	
	+19		-4	-3	+24			-5
		+2	+2		-8	-10	-6	
	+1		-4	-3				-5
		+2	+2					
Σ	+124	-62	+62	-6	-17	+30	-13	-60

Fig. B 5.1.2-1

B 5.1.2 Sample Problem (Cont'd)

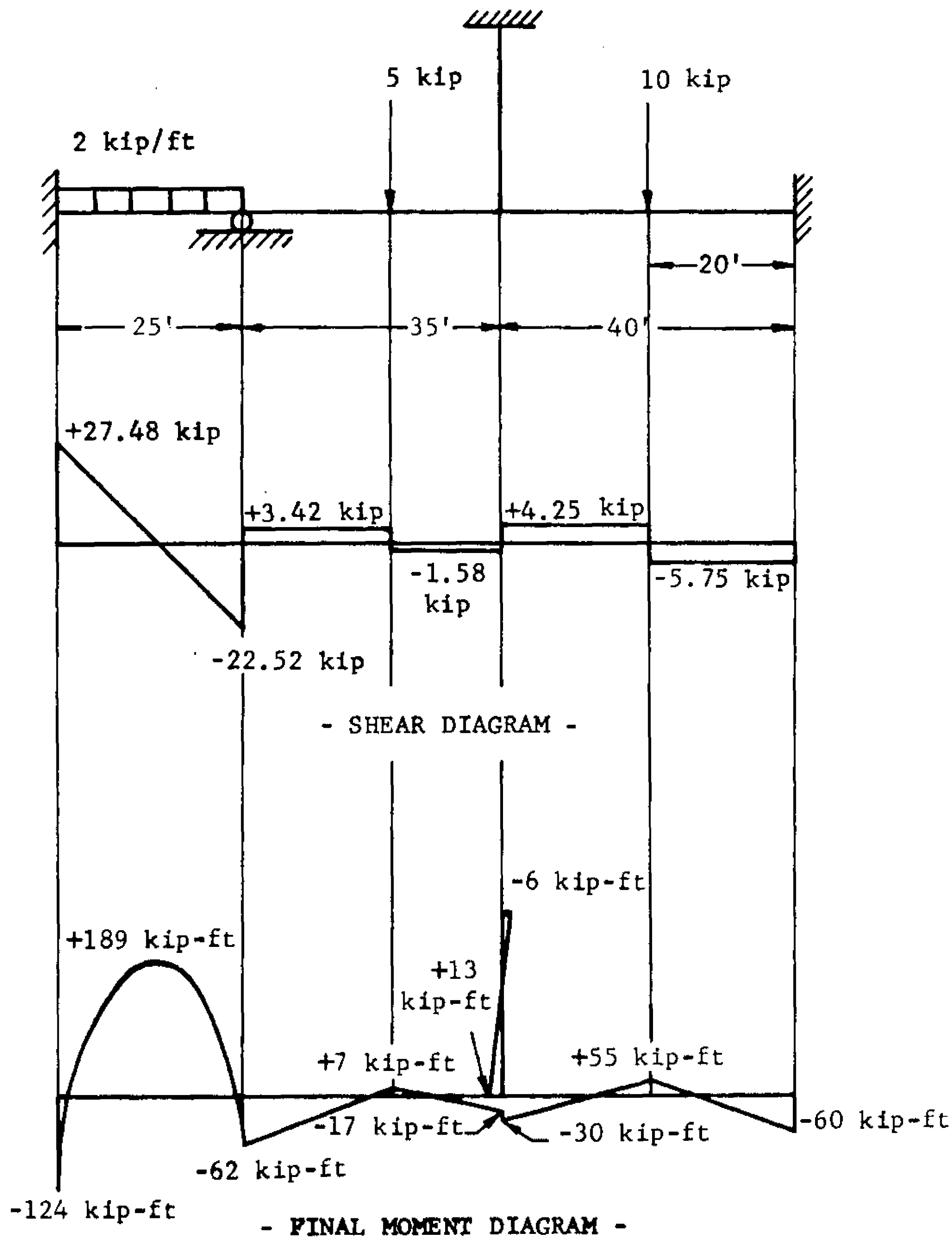


Fig. B 5.1.2-2

B 5.1.3 Application of Moment-distribution to Advanced Problems.

In article B 5.1.1, "Discussion of moment-distribution", the basic principles of moment-distribution were founded. Moment-distribution may be applied to complex structures involving joint translation, settlement of supports, non-prismatic members, symmetrical and unsymmetrical bents and other involved structures. For information on the technique of solving such structures see the references listed in Section B 5.0.0. Methods for accelerating the convergence and other short-cuts may also be obtained in these references.

B 5.1.4 Particular Solution of Bents and Semicircular Arches

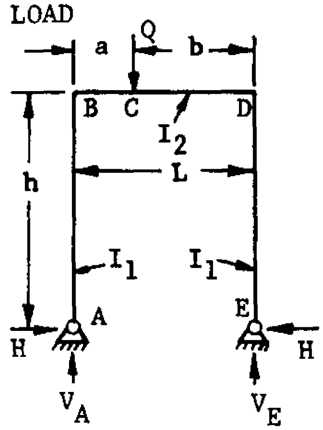
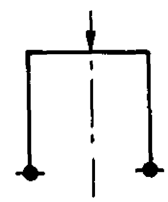
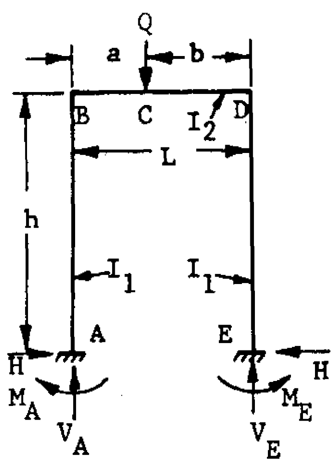
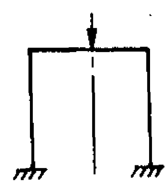
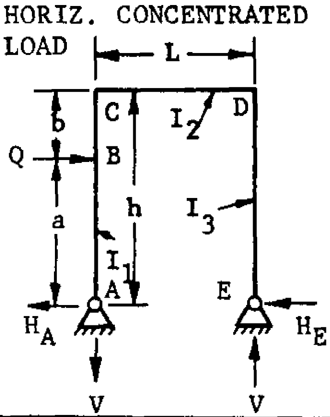
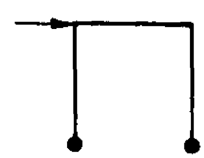
In the tables that follow, formulas for computing reactions are given for several load cases. In all cases constraining moments, reaction, and applied loads are positive when acting as shown and

$$K = \frac{I_2 h}{I_1 L} \quad \text{for cases 1 through 18}$$

$$K = \frac{I_1 S_2}{I_2 S_1} \quad \text{for cases 19 through 28}$$

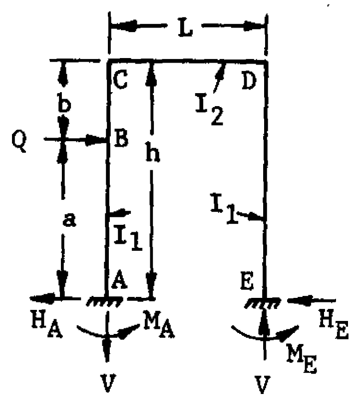
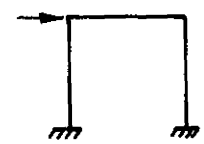
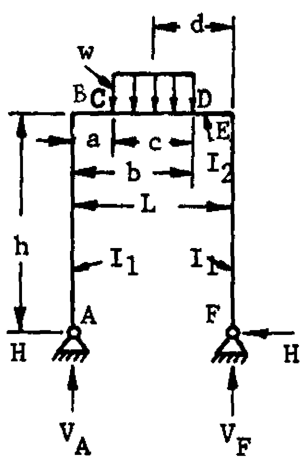
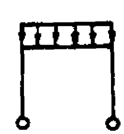
B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents

1. VERT. CONCENTRATED LOAD 	$V_A = \frac{Qb}{L}$ $\quad V_E = Q - V_A$ $H = \frac{3Qab}{2Lh(2K+3)}$ FOR SPECIAL CASE: $a = b = \frac{L}{2}$  $V_A = V_E = \frac{Q}{2}$ $H = \frac{3QL}{8h(2K+3)}$
2. VERT. CONCENTRATED LOAD 	$V_A = \frac{Qb}{L}\left[1 + \frac{a(b-a)}{L^2(6K+1)}\right]$ $\quad V_E = Q - V_A$ $H = \frac{3Qab}{2Lh(K+2)}$ $M_A = \frac{Qab}{L}\left[\frac{1}{2(K+2)} - \frac{(b-a)}{2L(6K+1)}\right]$ $M_E = \frac{Qab}{L}\left[\frac{1}{2(K+2)} + \frac{(b-a)}{2L(6K+1)}\right]$  FOR SPECIAL CASE: $a = b = \frac{L}{2}$ $V_A = V_E = \frac{Q}{2}$ $\quad M_A = M_E = \frac{QL}{8(K+2)}$
3. HORIZ. CONCENTRATED LOAD 	$V = \frac{Qa}{L}$ $\quad H_A = Q - H_E$ $H_E = \frac{Qa}{2h}\left[\frac{bK(a+h)}{h^2(2K+3)} + 1\right]$ FOR SPECIAL CASE: $b = 0,\ a = h$  $V = \frac{Qh}{L}$ $H_E = H_A = \frac{Q}{2}$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

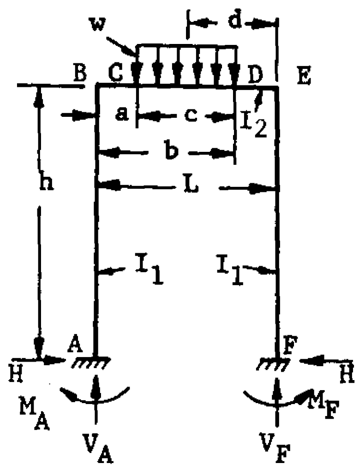
Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

4. HORIZ. CONCENTRATED LOAD  	$V = \frac{3Qa^2K}{Lh(6K+1)}$ $H_A = Q - H_E$ $H_E = \frac{Qab}{2h^2}\left[\frac{h}{b} - \frac{h+b+K(b-a)}{h(K+2)}\right]$ $M_A = \frac{Qa}{2h}\left[\frac{b(h+b+bK)}{h(K+2)} + h - \frac{3aK}{(6K+1)}\right]$ $M_E = \frac{Qa}{2h}\left[\frac{-b(h+b+bK)}{h(K+2)} + h - \frac{3aK}{(6K+1)}\right]$ FOR SPECIAL CASE: $b = 0,\ a = h$ $V = \frac{3QhK}{L(6K+1)}$ $H_A = H_E = \frac{Q}{2}$ $M_A = M_E = \frac{Qh(3K+1)}{2(6K+1)}$ 
5. VERT. UNIFORM RUNNING LOAD  $d = L - \frac{a}{2} - \frac{b}{2}$	$V_A = \frac{wcd}{L}$ $V_F = wc - V_A = \frac{wc}{L}\left(a + \frac{c}{2}\right) = wc\left(1 - \frac{d}{L}\right)$ $H = \frac{3}{2h}\left[\frac{X_1 + X_2}{2K+3}\right] = \frac{3wc}{24Lh(2K+3)}\left[12dL - 12d^2 - c^2\right]$ where: $X_1 = -\frac{wc}{24L}\left[24\frac{d^3}{L} - 6\frac{bc^2}{L} + 3\frac{c^2}{L} + 4c^2 - 24d^2\right]$ $X_2 = \frac{wc}{24L}\left[24\frac{d^3}{L} - 6\frac{bc^2}{L} + 3\frac{c^3}{L} + 2c^2 - 48d^2 + 24dL\right]$ FOR SPECIAL CASE: $a = 0,\ c = b = L,\ d = \frac{L}{2}$ $V_A = V_F = \frac{wL}{2}$ $H = \frac{wL^2}{4h(2K+3)}$ 

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

6. VERT. UNIFORM RUNNING LOAD



$$d = L - \frac{a}{2} - \frac{b}{2}$$

$$V_A = \frac{wcd}{L} + \frac{X_1 - X_2}{L(6K + 1)}$$

X_1 and X_2 are given in case 5

$$V_F = wc - V_A$$

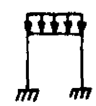
$$H = \frac{3(X_1 + X_2)}{2h(K + 2)} \qquad M_A = \frac{X_1 + X_2}{2(K + 2)} - \frac{X_1 - X_2}{2(6K + 1)}$$

$$M_F = \frac{X_1 + X_2}{2(K + 2)} + \frac{X_1 - X_2}{2(6K + 1)}$$

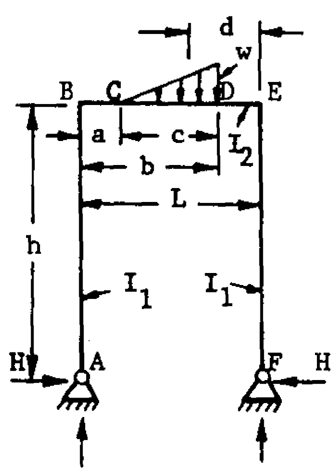
FOR SPECIAL CASE: $a = 0, \; c = b = L, \; d = \frac{L}{2}$

$$V_A = V_F = \frac{wL}{2}$$

$$H = \frac{wL^2}{4h(K + 2)} \qquad M_A = M_F = \frac{wL^2}{12(K + 2)}$$



7. VERT. TRIANGULAR RUNNING LOAD



$$d = L - \frac{a}{3} - \frac{2b}{3}$$

$$V_A = \frac{wcd}{2L}$$

$$V_F = \frac{wc}{2} - V_A = \frac{wc}{2L}\left(a + \frac{2c}{3}\right)$$

$$H = \frac{3}{2h}\left[\frac{X_3 + X_4}{2K + 3}\right] = \frac{3wc}{4Lh(2K + 3)}\left[dL - \frac{c^2}{18} - d^2\right]$$

WHERE:

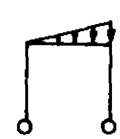
$$X_3 = -\frac{wc}{2L}\left[\frac{d^3}{L} + \frac{c^2}{9} + \frac{51c^3}{810L} + \frac{c^2 b}{6L} - d^2\right]$$

$$X_4 = \frac{wc}{2L}\left[\frac{d^3}{L} + \frac{c^2}{18} + \frac{51c^3}{810L} - \frac{c^2 b}{6L} - 2d^2 + dL\right]$$

FOR SPECIAL CASE: $a = o, \; c = b = L, \; d = \frac{L}{3}$

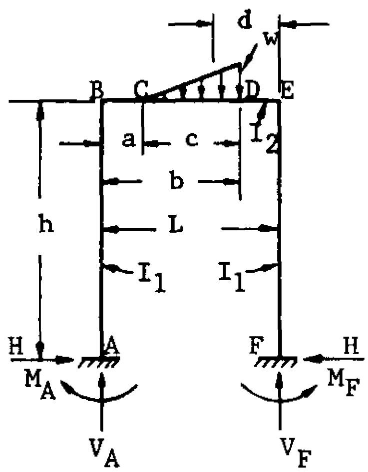
$$V = \frac{wL}{6}$$

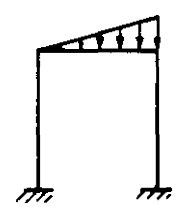
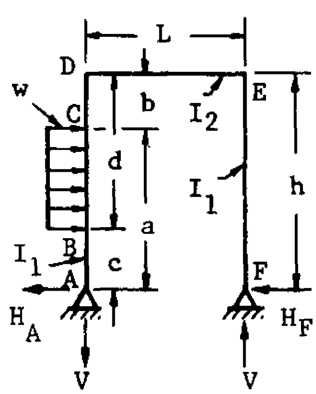
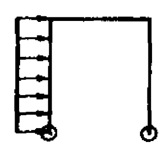
$$H = \frac{wL^2}{8h(2K + 3)}$$



B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

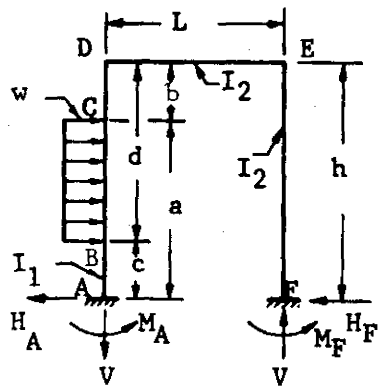
<table>
<tr>
<td>8. VERT. TRIANGULAR RUNNING LOAD


$d = L - \frac{a}{3} - \frac{2b}{3}$</td>
<td>$V_A = \frac{wcd}{2L} + \frac{X_3 - X_4}{L(6K + 1)}$ X_3 and X_4 are given in case 7
$V_F = \frac{wc}{2} - V_A$ $H = \frac{3(X_3 + X_4)}{2h(K + 2)}$
$M_A = \frac{X_3 + X_4}{2(K + 2)} - \frac{X_3 - X_4}{2(6K + 1)}$
$M_F = \frac{X_3 + X_4}{2(K + 2)} + \frac{X_3 - X_4}{2(6K + 1)}$
FOR SPECIAL CASE: a=o, c=b=L, $d = \frac{L}{3}$
$V_A = \frac{wL}{6}\left[1 - \frac{1}{10(6K + 1)}\right]$
$V_F = \frac{wL}{3}\left[1 + \frac{1}{20(6K + 1)}\right]$

$H = \frac{wL^2}{8h(K + 2)}$
$M_A = \frac{wL^2}{120}\left[\frac{5}{K + 2} + \frac{1}{6K + 1}\right]$
$M_F = \frac{wL^2}{120}\left[\frac{5}{K + 2} - \frac{1}{6K + 1}\right]$</td>
</tr>
<tr>
<td>9. HORIZ. UNIFORM RUNNING LOAD

</td>
<td>$V = \frac{w(a^2 - c^2)}{2L}$ $H_A = w(a - c) - H_F$
$H_F = \frac{w(a^2 - c^2)}{4h} + \frac{K\left[w(a^2-c^2)(2h^2-a^2-c^2)\right]}{8h^3(2K + 3)}$
FOR SPECIAL CASE: c=o, b=o, a=d=h
$V = \frac{wh^2}{2L}$

$H_A = wh - H_F$
$H_F = \frac{wh}{4}\left[1 + \frac{K}{2(2K + 3)}\right]$</td>
</tr>
</table>

B 5.1.4 <u>Particular Solution of Bents and Semicircular Arches (Cont'd)</u>

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

10. HORIZ. UNIFORM RUNNING LOAD



$$V = \frac{w(a^2 - c^2)}{2L} - \frac{M_A}{L} - \frac{M_F}{L}$$

$$H_A = w(a - c) - H_F$$

$$H_F = \frac{w(a^2 - c^2)}{4h} - \frac{X_5}{2h} + \frac{X_6(K - 1)}{2h(K + 2)}$$

WHERE:

$$X_5 = \frac{w}{12h^2}\left[d^3(4h - 3d) - b^3(4h - 3b)\right]$$

$$X_6 = \frac{w}{12h^2}\left[a^3(4h - 3a) - c^3(4h - 3c)\right]$$

$$M_A = \frac{(3K + 1)\left[\frac{w(a^2 - c^2)}{2} - X_5\right]}{2(6K + 1)} + \frac{X_6}{2}\left[\frac{1}{K + 2} + \frac{3K}{6K + 1}\right] + X_5$$

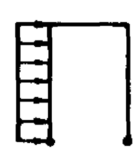
$$M_F = \frac{(3K + 1)\left[\frac{w(a^2 - c^2)}{2} - X_5\right]}{2(6K + 1)} - \frac{X_6}{2}\left[\frac{1}{K + 2} - \frac{3K}{6K + 1}\right]$$

<u>FOR SPECIAL CASE:</u> c=o, b=o, a=d=h:

$$V = \frac{wh^2K}{L(6K + 1)} \qquad H_A = wh - H_F$$

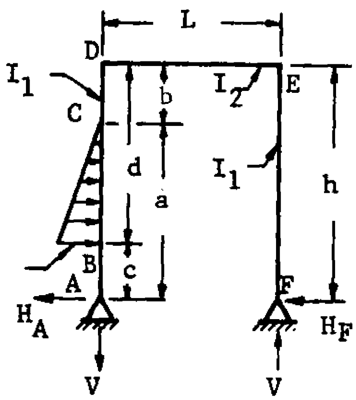
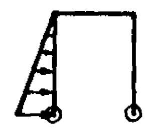
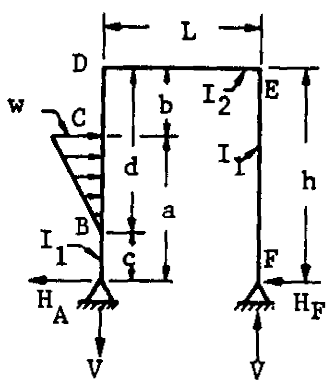
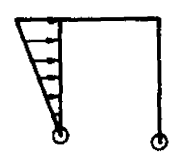
$$H_F = \frac{wh(2K + 3)}{8(K + 2)} \qquad M_F = \frac{wh^2}{24}\left[\frac{18K + 5}{6K + 1} - \frac{1}{K + 2}\right]$$

$$M_A = \frac{wh^2}{24}\left[\frac{30K + 7}{6K + 1} + \frac{1}{K + 2}\right]$$



B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

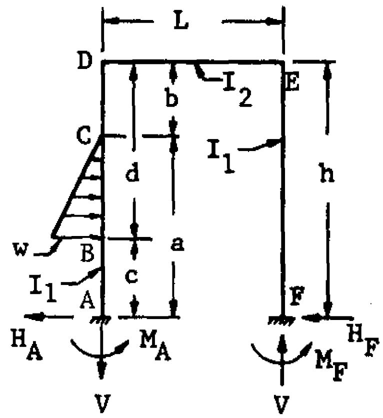
Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

11. HORIZ. TRIANGULAR RUNNING LOAD  	$V = \frac{w}{6L}(a^2 + ac - 2c^2)$ $H_A = \frac{w(a - c)}{2} - H_F$ $H_F = \frac{VL}{2h} + \frac{KX_7}{(2K + 3)h}$ WHERE: $X_7 = \frac{w}{120h^2(d-b)}\left[3(4d^5+b^5) - 15h(3d^4+b^4) + 20h^2(2d^3+b^3) - 15bd^2(2h-d)^2\right]$ FOR SPECIAL CASE: b=c=o, a=d=h: $V = \frac{wh^2}{6L}$ $H_A = \frac{wh}{2} - H_F$  $H_F = \frac{wh}{12}\left[1 + \frac{7K}{10(2K + 3)}\right]$
12. HORIZ. TRIANGULAR RUNNING LOAD  	$V = \frac{w}{6L}(2a + c)(a - c)$ $H_A = \frac{w(a - c)}{2} - H_F$ $H_F = \frac{VL}{2h} + \frac{KX_{10}}{h(2K + 3)}$ WHERE: $X_{10} = \frac{w}{120h^2(a-c)}\left[-30h^2c(a^2-c^2) + 20h^2(a^3-c^3) + 15c(a^4-c^4) - 12(a^5-c^5)\right]$ FOR SPECIAL CASE: b=c=o, a=d=h: $V = \frac{wh^2}{3L}$  $H_A = \frac{wh}{2} - H_F$ $H_F = \frac{wh}{10}\left[\frac{4K + 5}{2K + 3}\right]$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

13. HORIZ. TRIANGULAR RUNNING LOAD



$$V = \frac{w(a^2 + ac - 2c^2)}{6L} - \frac{M_A}{L} - \frac{M_F}{L}$$

$$H_A = \frac{w(a - c)}{2} - H_F$$

$$H_F = \frac{w(a^2 + ac - 2c^2)}{12h} - \frac{X_8}{2h} + \frac{X_9(K-1)}{2h(K+2)}$$

WHERE:

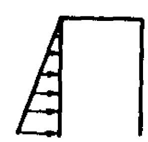
$$X_8 = \frac{w}{60h^2(d-b)}\left[15(h+b)(d^4-b^4) - 12(d^5-b^5) - 20bh(d^3-b^3)\right]$$

$$X_9 = \frac{w}{60h^2(d-b)}\left[10d^2h^2(2d-3b) + 10bh(4d^3+b^2h-b^3) - d^4(30h+15b) + 12d^5 + 3b^5\right]$$

$$M_A = \frac{(3K + 1)\left[\frac{w(a^2 + ac - 2c^2)}{6} - X_8\right]}{2(6K + 1)} + \frac{X_9}{2}\left[\frac{1}{K + 2} + \frac{3K}{6K + 1}\right] + X_8$$

$$M_F = \frac{(3K + 1)\left[\frac{w(a^2 + ac - 2c^2)}{6}\right] - X_8}{2(6K + 1)} - \frac{X_8}{2}\left[\frac{1}{K + 2} - \frac{3K}{6K + 1}\right]$$

FOR SPECIAL CASE: $b=c=o$, $a=d=h$



$$V = \frac{wh^2K}{4L(6K + 1)} \qquad H_A = \frac{wh}{2} - H_F$$

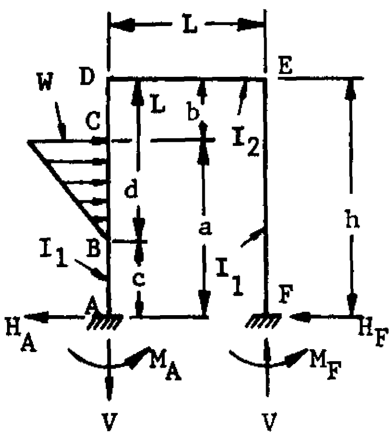
$$H_F = \frac{wh(3K + 4)}{40(K + 2)} \qquad M_F = \frac{wh^2}{60}\left[\frac{27K+7}{2(6K+1)} - \frac{1}{K+2}\right]$$

$$M_A = \frac{wh^2}{60}\left[\frac{27K + 7}{2(6K + 1)} + \frac{3K + 7}{K + 2}\right]$$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

14. HORIZ. TRIANGULAR RUNNING LOAD



$$V = \frac{w(2a + c)(a - c)}{6L} - \frac{M_A}{L} - \frac{M_F}{L}$$

$$H_A = \frac{w(a - c)}{2} - H_F$$

$$H_F = \frac{w(2a^2 - ac - c^2)}{12h} - \frac{X_{11}}{2h} + \frac{X_{12}(K - 1)}{2h(K + 2)}$$

where:

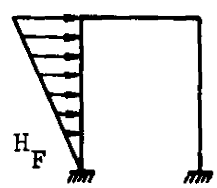
$$X_{11} = \frac{w}{60h^2(d-b)}\left[5hd^4 - 3d^5 - 20hdb^3 - 12b^4(d+h)\right]$$

$$X_{12} = \frac{w}{60h^2(a-c)}\left[15(h+c)(a^4 - c^4) - 12(a^5 - c^5) - 20ch(a^3 - c^3)\right]$$

$$M_A = \frac{\left[3K+1\right]\left[\frac{w(2a^2 - ac - c^2)}{6} - X_{11}\right]}{2(6K + 1)} + \frac{X_{12}}{2}\left[\frac{1}{K + 2} + \frac{3K}{6K + 1}\right] + X_{11}$$

$$M_F = \frac{\left[3K+1\right]\left[\frac{w(2a^2 - ac - c^2)}{6} - X_{11}\right]}{2(6K + 1)} - \frac{X_{22}}{2}\left[\frac{1}{K + 2} - \frac{3K}{6K + 1}\right]$$

FOR SPECIAL CASE: $b = c = o$, $a = d = h$



$$V = \frac{3Kwh^2}{4L(6K + 1)} \qquad H_A = \frac{wh}{2} - H_F$$

$$H_F = \frac{wh(7K + 11)}{40(K + 2)} \qquad M_A = \frac{wh^2}{120}\left[\frac{87K+22}{6K+1} + \frac{3}{K+2}\right]$$

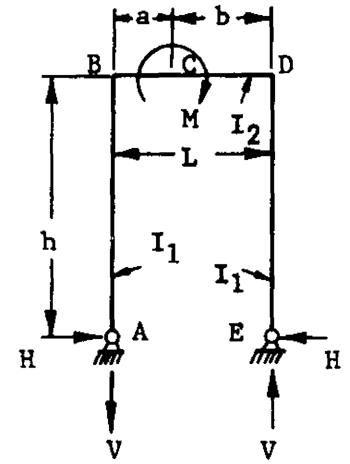
$$M_F = \frac{wh^2}{40}\left[\frac{21K + 6}{6K + 1} - \frac{1}{K + 2}\right]$$

B 5.1.4 <u>Particular Solution of Bents and Semicircular Arches (Cont'd)</u>

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

<table>
<tr>
<td>

15. MOMENT ON HORIZ. SPAN



</td>
<td>

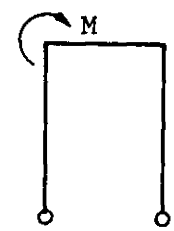
$$V = \frac{M}{L}$$

$$H = \frac{3(b - L/2)M}{Lh(2K + 3)}$$

<u>FOR SPECIAL CASE:</u> a=o, b=L

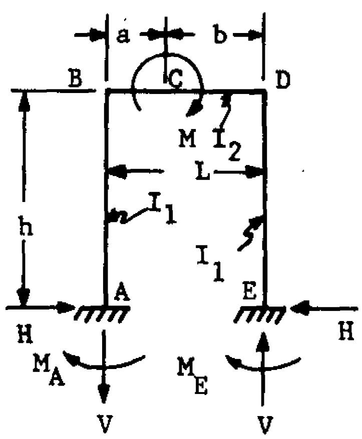
$$V = \frac{M}{L}$$

$$H = \frac{3M}{2h(2K + 3)}$$



</td>
</tr>
<tr>
<td>

16. MOMENT ON HORIZ. SPAN



</td>
<td>

$$V = \frac{6(ab + L^2K)M}{L^3(6K + 1)}$$

$$H = \frac{3(b - a)M}{2Lh(K + 2)}$$

$$M_A = M\left[\frac{6ab(K+2) - L\left[a(7K+3) - b(5K-1)\right]}{2L^2(K+2)(6K+1)}\right]$$

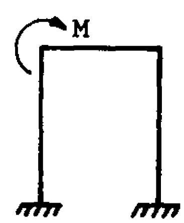
$$M_E = VL - M - M_A$$

<u>FOR SPECIAL CASE:</u> a=o, b=L

$$V = \frac{6KM}{L(1 + 6K)}$$

$$H = \frac{3M}{2h(K + 2)}$$

$$M_A = \frac{(5K - 1)M}{2(K + 2)(6K + 1)}$$



</td>
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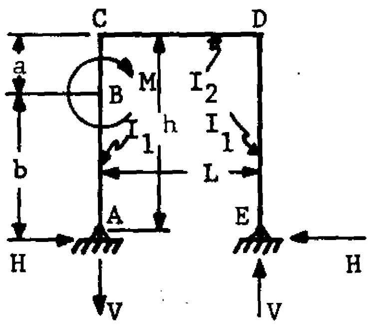
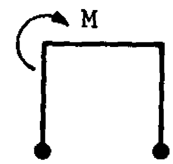
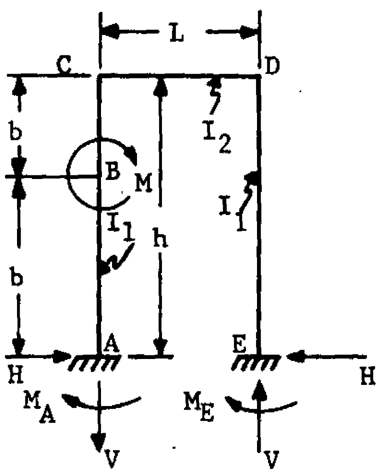
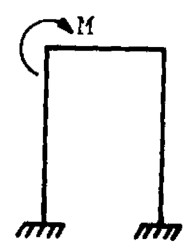
</table>

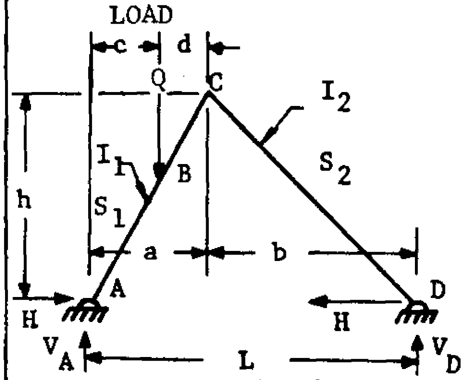
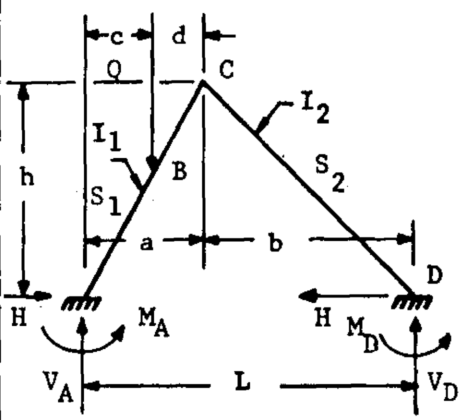
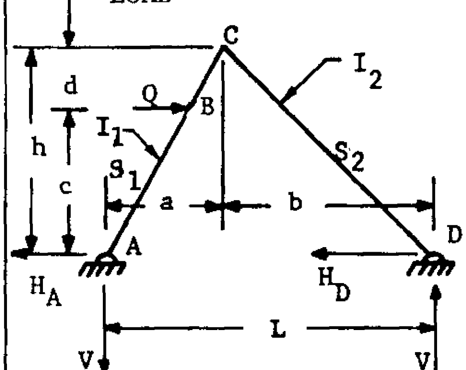
B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

17. MOMENT ON SIDE SPAN 	$V = \frac{M}{L}$ $\qquad H = \frac{3\left[K(2ab+a^2) + h^2\right]M}{2h^3(2K+3)}$ FOR SPECIAL CASE: a=o, b=h $V = \frac{M}{L}$ $H = \frac{3M}{2h(2K+3)}$ 
18. MOMENT ON SIDE SPAN 	$V = \frac{6bKM}{hL(6K+1)}$ $\qquad H = \frac{3bM\left[2a(K+1) + b\right]}{2h^3(K+2)}$ $M_A = \frac{-M}{2h^2(K+2)(6K+1)}\left[4a^2+2ab+b^2+K(26a^2-5b^2) + 6aK^2(2a-b)\right]$ $M_E = VL - M - M_A$ FOR SPECIAL CASE: a=o; b=h $V = \frac{6KM}{L(6K+1)}$ $H = \frac{3M}{2h(K+2)}$ $M_A = \frac{M(5K-1)}{2(K+2)(6K+1)}$ 

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

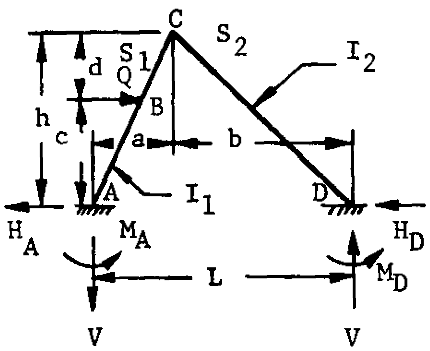
Table B 5.1.4-2 Reactions and Constraining Moments in Triangular Bents (Cont'd)

19. VERT. CONCENTRATED LOAD  	$V_A = Q - V_D$ $V_D = \frac{Qc}{L}$ $H = \frac{Qc}{h}\left[\frac{b}{L} + \frac{d(a+c)}{2a^2(K+1)}\right]$
20. VERT. CONCENTRATED LOAD  	$V_A = Q - V_D$ $V_D = \frac{Qc}{L}\left[1 - \frac{d(a+d)}{2a^2}\right]$ $H = \frac{Qcb}{Lh} + \frac{Qcd}{6La^2h(K+1)}\left\{\left[-b(3K+4) - 2L\right]\left[(a+d)\right] + 2(2L+b)(a+c) + 3ac\right\}$ $M_A = \frac{Qcd}{6a^2(K+1)}\left[(a+d)(3K+4) - 2(a+c)\right]$ $M_D = \frac{Qc^2d}{2a^2(K+1)}$
21. HORIZ. CONCENTRATED LOAD  	$V = \frac{Qc}{L}$ $H_A = Q - H_D$ $H_D = \frac{Qc}{h}\left[\frac{b}{L} + \frac{d(h+c)}{2h^2(K+1)}\right]$

B 5.1.4 <u>Particular Solution of Bents and Semicircular Arches (Cont'd)</u>

Table B 5.1.4-2 Reactions and Constraining Moments in Triangular Bents (Cont'd)

22. HORIZ. CONCENTRATED LOAD



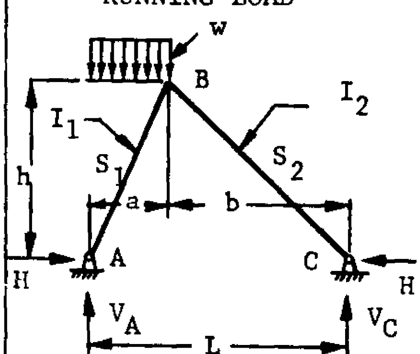
$$V = \frac{Qc}{L}\left[1 - \frac{d(h+d)}{2h^2}\right]; \qquad H_A = Q - H_D$$

$$H_D = \frac{Qc}{Lh}\left\{b + \frac{d}{6h^2(K+1)}\left[(h+d)(-b[3K+4] - 2L) + 2(2L+b)(h+e) + 3ac\right]\right\}$$

$$M_A = \frac{Qcd}{6h^2(K+1)}\left[(h+d)(3K+4) - 2(h+c)\right]$$

$$M_D = \frac{Qcd}{6h^2(K+1)}(h + 2c + d)$$

23. VERTICAL UNIFORM RUNNING LOAD

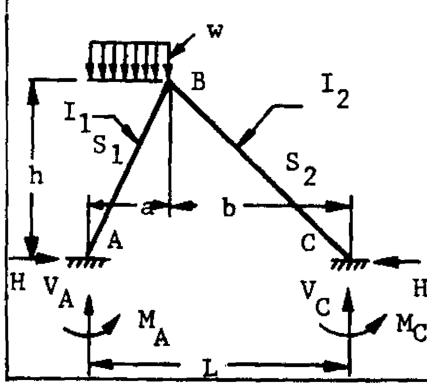


$$V_A = wa\left[1 - \frac{a}{2L}\right]$$

$$V_C = \frac{wa^2}{2L}$$

$$H = \frac{wa^2}{8h}\left[\frac{4b}{L} + \frac{1}{1+K}\right]$$

24. VERTICAL UNIFORM RUNNING LOAD



$$V_A = wa\left[1 - \frac{3a}{8L}\right] \quad : \quad V_C = \frac{3wa^2}{8L}$$

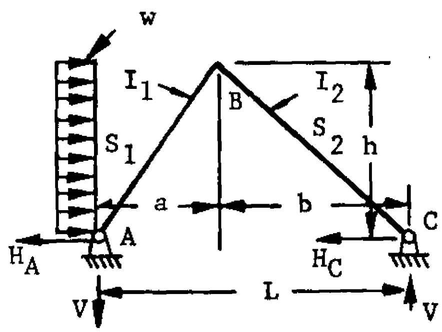
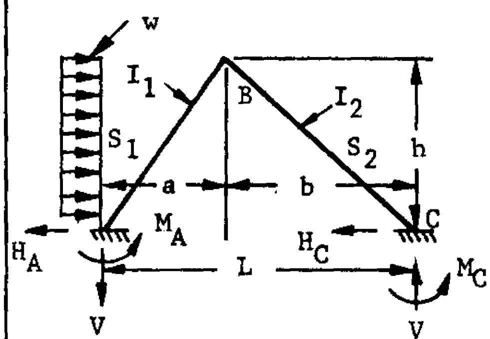
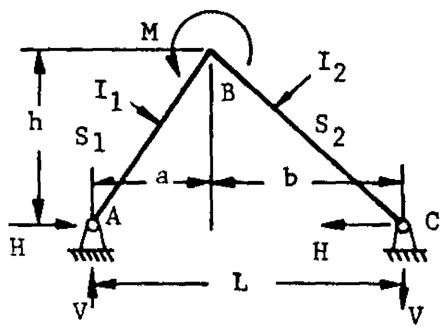
$$H = \frac{wa^2}{24Lh(K+1)}\left[b(10 + 9K) + 2L + a\right]$$

$$M_A = \frac{wa^2(3K+2)}{24(K+1)}$$

$$M_c = \frac{wa^2}{24(K+1)}$$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-2 Reactions and Constraining Moments in Triangular Bents (Cont'd)

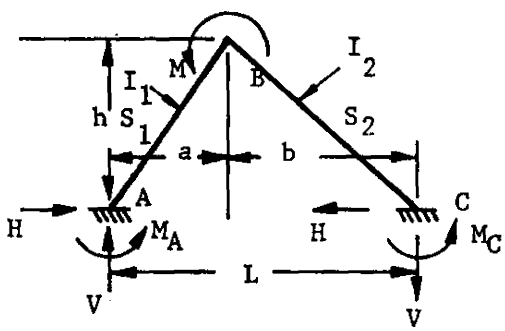
25. HORIZ. UNIFORM RUNNING LOAD 	$V = \frac{wh^2}{2L}$ $H_A = wh - H_C$ $H_C = \frac{wh}{8}\left[\frac{4b}{L} + \frac{1}{K+1}\right]$
26. HORIZ. UNIFORM RUNNING LOAD 	$V = \frac{3wh^2}{8L}$ $M_A = wh - M_C$ $M_C = \frac{wh}{8L(K+1)}\left[b(3K+4) + a\right]$ $M_A = \frac{wh^2(3K+2)}{24(K+1)}$ $M_C = \frac{wh^2}{24(K+1)}$
27. APPLIED MOMENT AT APEX 	$V = \frac{M}{L}$ $H = \frac{M}{hL}\left[\frac{a - bK}{K+1}\right]$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-3 Reactions and Constraining Moments in Triangular Bents and Semicircular Frames or Arches (Cont'd)

<table>
<tr><td>

28. APPLIED MOMENT AT APEX



</td><td>

$V = \frac{3M}{2L}$

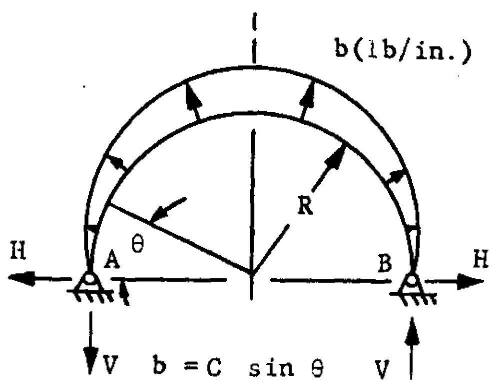
$H = \frac{3M(a - bK)}{2hL(K + 1)}$

$M_A = \frac{KM}{2(K + 1)}$

$M_C = \frac{M}{2(K + 1)}$

</td></tr>
<tr><td>

29. SINUSOIDAL NORMAL PRESSURE



</td><td>

$V = \frac{C\pi R}{4}$

$H = \frac{CR}{4}$

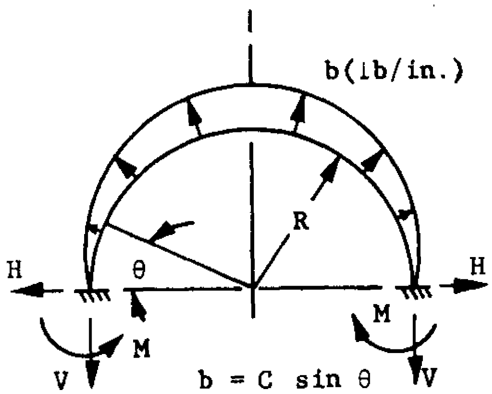
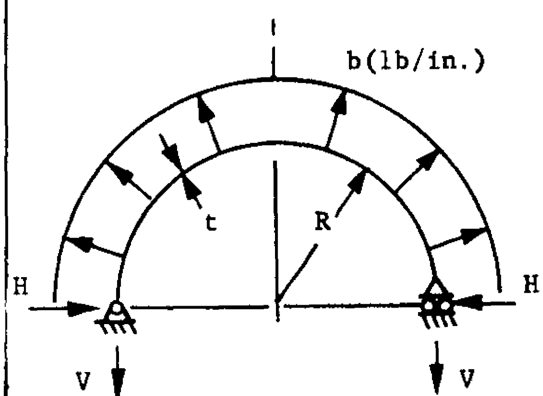
$M_\theta = \frac{CR^2}{4} \left[(\pi - 2\theta)\cos\theta - \pi + 3\sin\theta \right]$

(Positive moment acts clockwise on section ahead.)

</td></tr>
</table>

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4 Reactions and Constraining Moments in Semicircular Frames or Arches (Cont'd)

30. SINUSOIDAL NORMAL PRESSURE  	$V = \frac{C\pi R}{4}$ $H = \frac{CR}{4}\left[\frac{3\pi^2 - 32}{8 - \pi^2}\right] = .31974CR$ $M = \frac{CR^2}{4}\left[\frac{\pi^3 - 10\pi}{8 - \pi^2}\right] = .05478CR^2$ $M_\theta = CR^2\left[.81974 \sin\theta - .84018 + \frac{\cos\theta}{2}\left(\frac{\pi}{2} - \theta\right)\right]$ (Positive moment acts clockwise on section ahead.)
31. UNIFORM NORMAL PRESSURE  	M = 0 at all points since pin points permit a uniform hoop tension. T, where: $T = V = bR$ $H = 0$

B5.2.0 Analysis of Arbitrary Ring by Tabular Method

The following analysis is a corrected version of work taken originally from Reference 6.

Rings or frames with ends built in, elastically restrained, or pinned may be analyzed by the procedure outlined in this section. The procedure is given in tabular form (Table B5.2.0-1) with added notes to define the sequence to be followed.

A sample problem is given in Sec. B5.2.1 to illustrate the procedure. This sample problem considers the energy due to direct and shear loads. The effects of shear flow are presented as a supplement to the sample problem.

Procedure

I. Procedure to Obtain Initial Geometric and Elastic Data

(1) Set out the neutral axis of the ring between the end points T and O. (In a complete ring T and O coincide.)

(2) Divide the neutral axis into a number of segments which are conveniently but not necessarily of equal length Δs. Ten to twenty segments will usually give sufficient accuracy. Mark the joints and central points of the segments. In symmetrical rings a complete segment should lie each side of the axis of symmetry.

(3) Calculate the values $\Delta s/EI$, $\Delta s/EA$, $\Delta s/GA'$ at the segment centers.

(4) For a built-in ring, calculate a and b from equation (1) below which defines the elastic center C and the axes X' and Y' (See Figures B5.2.0-1 and -2.)

For a pinned-end ring, no translation of axes from O is necessary. Use the X and Y axes as they are defined in Figures B5.2.0-3 and -4,

(5) For a built-in ring obtain the co-ordinates x', y', and the angles ψ' at the segment centers. For a pinned ring obtain x, y, and ψ directly. In symmetrical rings only half the ring need be considered.

B 5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

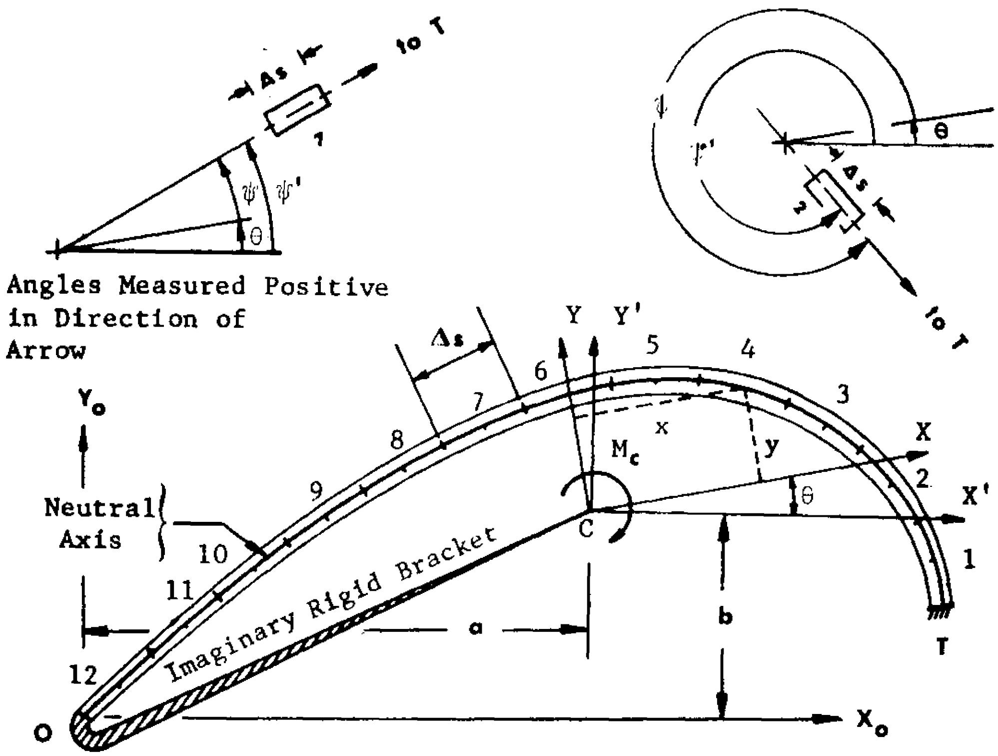


Fig. B5.2.0-1 General Ring with Built-In Ends

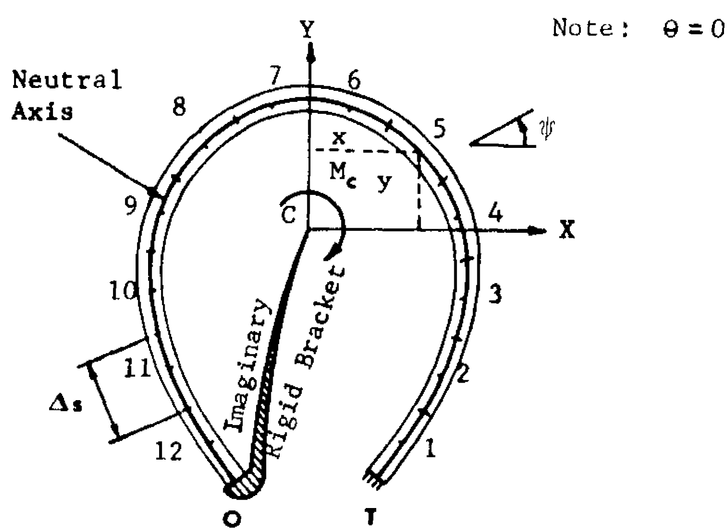


Fig. B5.2.0-2 Symmetrical Built-In Ring

B 5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

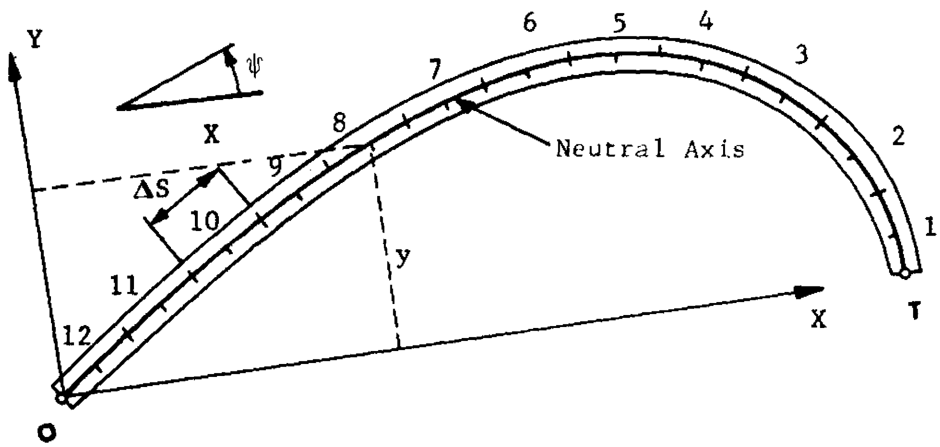


Fig. B 5.2.0-3 General Ring with Pinned End

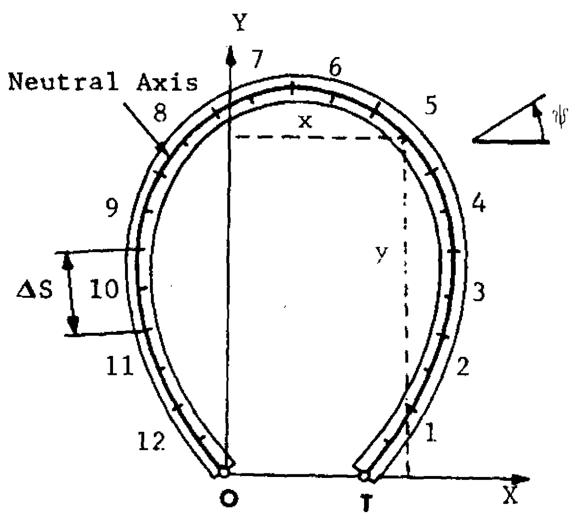


Fig. B 5.2.0-4 Symmetrical Pinned-End Ring

(6) Note that ψ, θ, and ψ' are measured as shown in Figure B5.2.0-1. The angle must originate at the end coming from O, and be measured counterclockwise to the end going to T.

For the slope of the tangent to the neutral axis of the element, use the slope at the midspan of the element.

NOTE: $\Delta s/EA$ and $\Delta s/GA'$ are not usually required, since in most cases the thrust and shear energies are negligible compared with the bending energy.

General Notes

I. Limits of Application

The method may be applied to any ring or curved beam in which the deflections are linear functions of the loads and in which the elementary formula connecting curvature and bending moment holds.

II. Calculations

(1) The advantage of choosing X, Y, and M_c as redundant reactions is due to their orthogonality. Thus, one can calculate them directly without having to solve simultaneous equations.

(2) The calculations are shown in tabular form using finite segments. Graphical integration may also be used and in exceptional cases an analytical method may be applied.

(3) The referenced table is set out for an arbitrary built-in ring, but is also applicable when the supports T and O "give" elastically (See General Note III, 6.) In all other cases the size of the table is reduced but the form is essentially the same. A considerable reduction in the numerical work is possible when the ring is symmetrical (See notes IV and VI.)

(4) In practice certain steps of the calculation may be conveniently done with additional columns, e.g., for x and y one requires x' cos θ, etc.

(5) Numerical Accuracy. Although the data in columns 1-6 and 24-26 will not exceed three-figure accuracy it is necessary to retain four or five figures in the remaining columns in order that the final results shall be correct to three figures.

III. Arbitrary Built-In Ring (Fig. B 5.2.0-1)

Equations required and the stage at which they are used are as follows:

(1) $$a = \frac{\sum \frac{x_o \Delta s}{EI}}{\sum \frac{\Delta s}{EI}}$$

$$b = \frac{\sum \frac{y_o \Delta s}{EI}}{\sum \frac{\Delta s}{EI}}$$

(2) After column 12,

$$\tan 2\theta = \frac{2\Sigma x' y' \frac{\Delta s}{EI} - \Sigma \sin 2\psi' \left(\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right)}{\Sigma (x'^2 - y'^2) \frac{\Delta s}{EI} - \Sigma \cos 2\psi' \left(\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right)}$$

$x = x' \cos \theta + y' \sin \theta$; $y = y' \cos \theta - x' \sin \theta$.

(3) M_o, N_o, and S_o are calculated for the ring built in at **T** and free at O.

(4) After column 33,

$$\dot{X} = - \frac{-\Sigma M_o y \frac{\Delta s}{EI} + \Sigma N_o \cos \psi \frac{\Delta s}{EA} + \Sigma S_o \sin \psi \frac{\Delta s}{GA'}}{\Sigma y^2 \frac{\Delta s}{EI} + \Sigma \cos^2 \psi \frac{\Delta s}{EA} + \Sigma \sin^2 \psi \frac{\Delta s}{GA'}}$$

B 5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

$$Y = - \frac{\Sigma M_o x \frac{\Delta s}{EI} + \Sigma N_o \sin \psi \frac{\Delta s}{EA} - \Sigma S_o \cos \psi \frac{\Delta s}{GA'}}{\Sigma x^2 \frac{\Delta s}{EI} + \Sigma \sin^2 \psi \frac{\Delta s}{EA} + \Sigma \cos^2 \psi \frac{\Delta s}{GA'}}$$

$$M_c = - \frac{\Sigma M_o \frac{\Delta s}{EI}}{\Sigma \frac{\Delta s}{EI}}$$

(5) After column 39,

$$M = M_c - Xy + Yx + M_o$$

$$N = N_o + X \cos \psi + Y \sin \psi$$

$$S = S_o + X \sin \psi - Y \cos \psi$$

(6) Effect of Elastic Supports. The elastic characteristics at the supports T and O are represented by the two sets of three coefficients, k_Tm, k_Tn, k_Ts; k_Om, k_On, k_Os. Thus k_Tm is the moment required to rotate T through one radian, k_Tn is the force required to move T in a direction defined by ψ'_T through unit distance, and k_Ts is the force required to move T in a direction normal to ψ'_T through unit distance. At each support the rotation and the two displacements are assumed elastically orthogonal, i.e., when a force is applied to the support T in the direction ψ'_T, there is no rotation and no movement normal to ψ'_T.

For the purposes of the calculations the support flexibilities may be represented by two additional segments considered concentrated at T and O, of which the "elastic weights" are respectively:

$$\frac{\Delta s}{EI} = \frac{1}{k_T m}, \quad \frac{\Delta s}{EA} = \frac{1}{k_T n}, \quad \frac{\Delta s}{GA'} = \frac{1}{k_T s};$$

$$\frac{\Delta s}{EI} = \frac{1}{k_O m}, \quad \frac{\Delta s}{EA} = \frac{1}{k_O n}, \quad \frac{\Delta s}{GA'} = \frac{1}{k_O s}$$

B 5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

Note that ψ'_T and ψ'_O need not necessarily be the values of ψ' at T and O. The values above and coordinates are entered in rows T and O of the table.

IV. Symmetrical Built-In Ring (Fig. B 5.2.0-2)

(1) CY is the line of symmetry and columns 4-12 are not required as X, Y, and ψ are calculated directly.

(2) Any loading may be analyzed as a symmetrical and an anti-symmetrical loading.

(3) For symmetrical loading Y is statically determinate, being half the total load in the direction YC and should hence be included in the external loading. Thus in the table, columns 18, 20, 23, 28, 30, 33, 35, 38, and 39 are not required, and in the expressions for M, N, S terms involving Y are omitted. X and M_c are determined from the formulas in III (4).

(4) For antisymmetrical loading X and M_c are statically determinate being half the total load in the direction XC and equal and opposite to half the moment of the external load about C respectively; they should hence be included in the external loading. Thus, in the table, columns 19, 21, 22, 27, 29, 31, 32, 34, 36, and 37 are not required, and in the expressions for M, N, S the terms involving M_c and X are omitted. Y is determined from the formula in III (4).

(5) Note that only half the ring need be considered in the table, i.e., elements 7-12 and support O. For symmetrical loading M and N are symmetrical and S antisymmetrical. For anti-symmetrical loading M and N are antisymmetrical and S symmetrical. The summations are, of course, only to be taken over half the ring.

(6) When the supports have symmetrical elastic characteristics the method of this section still applies by introducing an additional segment at O, as described in III (6).

V. Arbitrary Pinned Ring (Fig. B 5.2.0-3)

(1) Columns 4, 12, 18, 20, 23, 27, 28, 30, 33, 35, 38, and 39 are not required.

(2) Y is statically determinate and should hence be included in the external loading.

(3) X is determined from the formula in III (4).

B5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

(4) The formulas for M, N, and S are:

$M = M_o - Xy$; $N = N_o + X \cos \psi$; $S = S_o + X \sin \psi$.

(5) Effect of Elastic Supports. In this case the procedure of III (6) should be applied, noting that the terms k_{Tm}, k_{Om} do not exist.

VI. Symmetrical Pinned Ring (Fig. B5.2.0-4)

(1) Any loading may be analyzed as a symmetrical and an antisymmetrical loading.

(2) For symmetrical loading, the load Y at O is obviously half the total load in the direction Y, and only half the ring need be considered in the table, i.e., elements 7-12 and support O. Otherwise, the procedure is the same as in the general case.

(3) For antisymmetrical loading X is statically determinate, being half the total load in the direction X. Thus, in this case the problem is solved purely by statics.

(4) When the supports have symmetrical elastic characteristics the method of this section still applies. For the symmetrical loading an additional segment should be introduced at O, as described in IV (6) and V (5).

Notation

A = cross-sectional area

A' = effective area of cross section for shear stiffness

C = elastic center = centroid of elastic weights $\Delta s/EI$

E = Young's modulus

G = shear modulus

I = moment of inertia of cross section

k = elastic constants of supports T and O (see note III,6)

M_c = moment at elastic center in fixed ring

M_o, N_o, S_o = bending moment, direct load, and shear load in ring supported at T, due to external loading and any of the reactions X, Y, and M_c, which may be statically determined (e.g., in a pinned ring Y is always statically determined). M_o is positive for compression in outer fibers. N_o is positive for compression and S_o is positive when acting outwards on the right-hand side of a section (Subscripts do not refer to point "O")

M, N, S = total bending moment, direct load, and shear load at any section. Sign convention as for M_o, N_o, S_o

O = origin, or left-hand end point of ring

T = terminus, or right-hand end point of ring or cross-sectional area of ring

Δs = length of segment

x_o, y_o = co-ordinates referred to arbitrary orthogonal axes OX', OY' of built-in ring

ψ' = angle defining slope of neutral axis of built-in ring, with respect to CX'

x, y = co-ordinates referred to geometric and elastic orthogonal axes CX, CY of built-in ring; or, co-ordinates referred to orthogonal axes OX, OY of pinned ring, where OX passes through T

θ = angle between CX' and CX

ψ = angle defining slope of neutral axis for built-in ring, with respect to CX ($\psi = \psi' - \theta$)

X, Y = reactions in directions CX, CY at elastic center for built-in ring, or in directions OX, OY for pinned ring

x',y' = coordinates referred to geometric and elastic orthogonal axes CX' and CY' of built-in ring

TABLE B 5.2.0-1

	1	2	3	4	5	6	7	8	9	10	11	12
Segments or Point	$\frac{\Delta s}{EI}$	$\frac{\Delta s}{EA}$	$\frac{\Delta s}{GA'}$	x'	y'	ψ'	$\sin 2\psi'$	$\cos 2\psi'$	$x'y'\frac{\Delta s}{EI}$	$[x'^2 - y'^2]\frac{\Delta s}{EI}$	$\sin 2\psi'\left[\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right]$	$\cos 2\psi'\left[\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right]$
	Columns 1 to 8 give properties of ring. Operations in this row described in terms of the column numbers								4x5x1	(4^2-5^2) x1	(2-3) x7	(2-3) x8
T												
1												
2												
3												
		Can usually be omitted (see cols. 20-23)									Can usually be omitted (see cols. 20-23)	
10												
11												
12												
0												
	Σ	The symbol Σ under a column denotes that the column should be summed							Σ	Σ	Σ	Σ

TABLE B 5.2.0-1 (Cont'd)

	13	14	15	16	17	18	19	20	21	22	23
Segments or Point	x	y	ψ	$\sin \psi$	$\cos \psi$	$x^2 \frac{\Delta s}{EI}$	$y^2 \frac{\Delta s}{EI}$	$\sin^2 \psi \frac{\Delta s}{EA}$	$\cos^2 \psi \frac{\Delta s}{EA}$	$\sin^2 \psi \frac{\Delta s}{GA'}$	$\cos^2 \psi \frac{\Delta s}{GA'}$
	See Note III (2)		6-θ			13^2 x1	14^2x1	16^2x2	17^2x2	16^2x3	17^2x3
T											
1											
2											
3											
								Omit these columns when energy due to direct and shear loads is negligible.			
10											
11											
12											
0											
						Σ	Σ	Σ	Σ	Σ	Σ

TABLE B 5.2.0-1 (Cont'd)

	24	25	26	27	28	29	30	31	32	33
Segments or Point	M_o	N_o	S_o	$M_o \frac{\Delta s}{EI}$	$M_o x \frac{\Delta s}{EI}$	$M_o y \frac{\Delta s}{EI}$	$N_o \sin \psi \frac{\Delta s}{EA}$	$N_o \cos \psi \frac{\Delta s}{EA}$	$S_o \sin \psi \frac{\Delta s}{GA'}$	$S_o \cos \psi \frac{\Delta s}{GA'}$
	See Note III (3)			24x1	24x13x1	24x14x1	25x16x2	25x17x2	26x16x3	26x17x3
T										
1										
2										
3										
							Omit these columns when energy due to direct and shear loads is negligible			
10										
11										
12										
0										
				Σ	Σ	Σ	Σ	Σ	Σ	Σ

TABLE B5.2.0-1 (Concluded)

	34	35	36	37	38	39	40	41	42
Segments or Point	Xy	Yx	X sin ψ	X cos ψ	Y sin ψ	Y cos ψ	M	N	S
	Xx14	See Note III (5) Yx13	Xx16	Xx17	Yx16	Yx17	M_c+24 -34+35	25+37 +38	26+36 -39

B5.2.1 Sample Problem

The frame to be analyzed in this sample problem is the symmetrical built-in ring illustrated in Figure 5.2.1-1. The ring is divided into 24 segments for this problem. The section properties at the center of each segment are used as average values for the entire segment. A typical cross section of the ring is shown in Figure 5.2.1-2.

The necessary calculations to determine the magnitudes of M, N, and S at each segment are shown in tabular form on pages 42 through 46 . The results have been plotted on graphs, which are shown on pages 47 through 49 .

The problem data, statement, and conditions follow.

Given:

$P = 100$ lb

$Q = 100$ lb

$M = 1000$ in.-lb

$R_o = 50$ in.

$R_a = 47$ in.

$E = 10 \times 10^6$ psi

$G = 3.85 \times 10^6$ psi

$A = h \times 1$

$A' = 5/6\ A$ (See Ref. 7)

Problem:

Determine the bending moment M, direct load N, and shear load S for the given conditions by use of Sec. B5.2.0.

B5.2.1 Sample Problem (Cont'd)

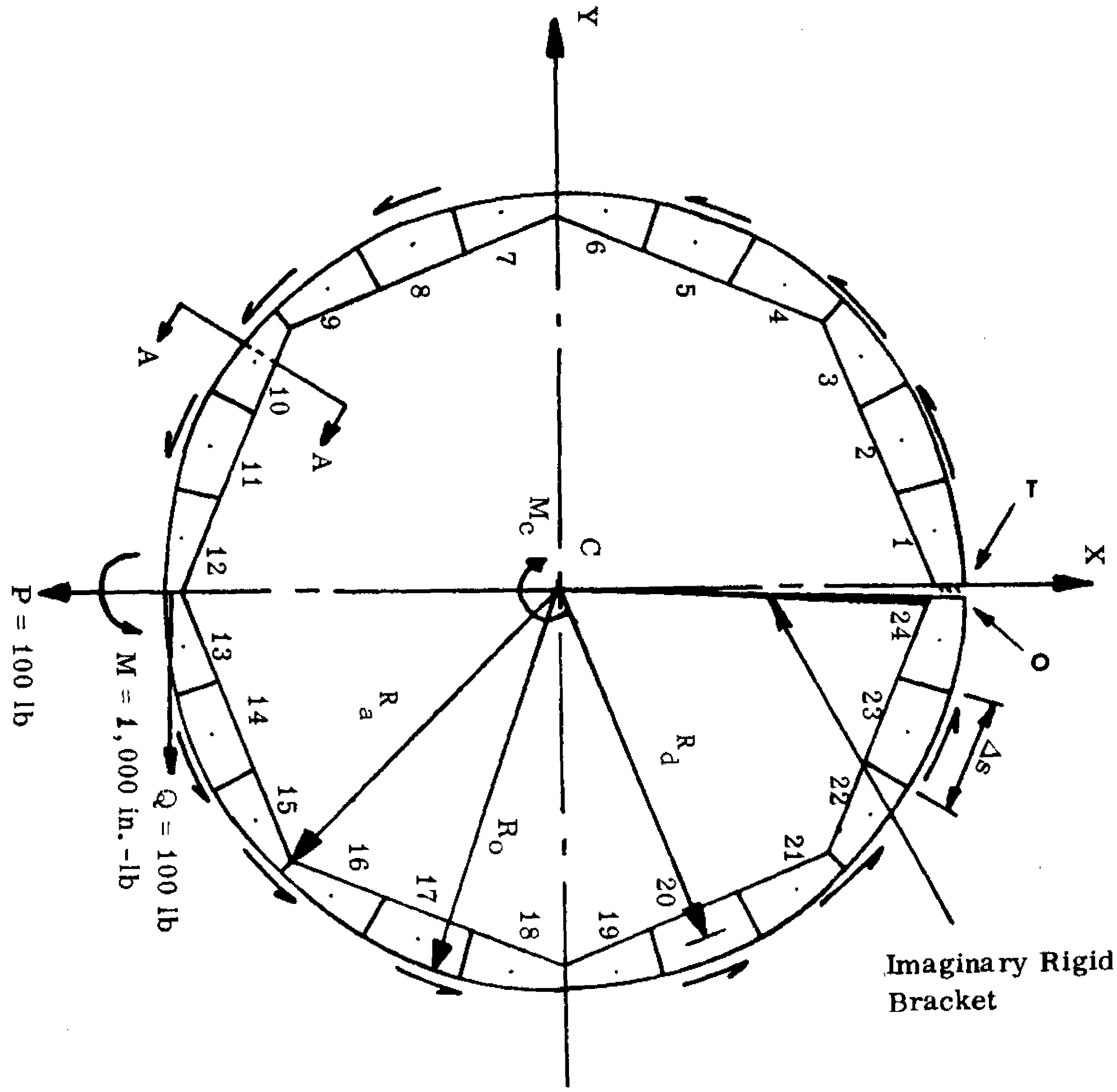


Figure B5.2.1-1 Frame for Sample Problem

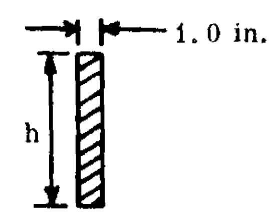


Figure B5.2.1-2 Typical Frame Cross Section

B5.2.1 Sample Problem (Cont'd)

	1	2	3	4	5	6	7	8
Segment or Points	$\frac{\Delta s}{EI}$	$\frac{\Delta s}{EA}$	$\frac{\Delta s}{GA'}$	x'	y'	ψ'	$\sin 2\psi'$	$\cos 2\psi'$
	Operations in this row described in terms of column numbers							
T	10^{-6}	10^{-6}	10^{-6}					
1	.11608	.24631	.76774					
2	.05156	.18591	.57946					
3	.11608	.24631	.76774					
4	.11608	.24631	.76774					
5	.05156	.18591	.57946					
6	.11608	.24631	.76774					
7	.11608	.24631	.76774					
8	.05156	.18591	.57946					
9	.11608	.24631	.76774					
10	.11608	.24631	.76774					
11	.05156	.18591	.57946					
12	.11608	.24631	.76774	Columns 4 through 12 not required for a symmetrical built-in ring;				
13	.11608	.24631	.76774					
14	.05156	.18591	.57946					
15	.11608	.24631	.76774					
16	.11608	.24631	.76774					
17	.05156	.18591	.57946					
18	.11608	.24631	.76774					
19	.11608	.24631	.76774					
20	.05156	.18591	.57946					
21	.11608	.24631	.76774					
22	.11608	.24631	.76774					
23	.05156	.18591	.57946					
24	.11608	.24631	.76774					
0								
Σ	2.26979	Sum of Columns						

B5.2.1 Sample Problem (Cont'd)

9	10	11	12	13	14	15	16	17
$x'y'\frac{\Delta s}{EI}$	$\left[x'^2 - y'^2\right]\frac{\Delta s}{EI}$	$\sin 2\psi'\left[\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right]$	$\cos 2\psi'\left[\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right]$	x	y	ψ	$\sin\psi$	$\cos\psi$
4×5×1	(4^2-5^2) ×1	(2-3) ×7	(2-3) ×8					
				See Note IV(1)				
				47.070	6.197	277.5	-.99144	.13051
				43.155	17.875	292.5	-.92387	.38267
				37.666	28.902	307.5	-.79334	.60875
	Columns 4 through 12 not required for a symmetrical built-in ring.			28.902	37.666	322.5	-.60875	.79334
				17.875	43.155	337.5	-.38268	.92387
				6.197	47.070	352.5	-.13051	.99144
				- 6.197	47.070	7.5	.13051	.99144
				-17.875	43.155	22.5	.38267	.92387
				-28.902	37.666	37.5	.60875	.79334
				-37.666	28.902	52.5	.79334	.60875
				-43.155	17.875	67.5	.92387	.38267
				-47.070	6.197	82.5	.99144	.13051
				-47.070	-6.197	97.5	.99144	-.13051
				-43.155	-17.875	112.5	.92387	-.38267
				-37.666	-28.902	127.5	.79334	-.60875
				-28.902	-37.666	142.5	.60875	-.79334
				-17.875	-43.155	157.5	.38267	-.92387
				- 6.197	-47.070	172.5	.13051	-.99144
				6.197	-47.070	187.5	-.13051	-.99144
				17.875	-43.155	202.5	-.38268	-.92387
				28.902	-37.666	217.5	-.60875	-.79334
				37.666	-28.902	232.5	-.79334	-.60875
				43.155	-17.875	247.5	-.92387	-.38267
				47.070	- 6.197	262.5	-.99144	-.13051

B5.2.1 Sample Problem (Cont'd)

18	19	20	21	22	23	24	25	26
$x^2 \frac{\Delta s}{EI}$	$y^2 \frac{\Delta s}{EI}$	$\sin^2 \psi \frac{\Delta s}{EA}$	$\cos^2 \psi \frac{\Delta s}{EA}$	$\sin^2 \psi \frac{\Delta s}{GA'}$	$\cos^2 \psi \frac{\Delta s}{GA'}$	M_o	N_o	S_o
$13^2 \times 1$	$14^2 \times 1$	$16^2 \times 2$	$17^2 \times 2$	$16^2 \times 3$	$17^2 \times 3$	See Note III(3) *		
10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}			
257.19	4.46	.24212	.00420	.75466	.01308	-11.7	.246	.320
96.02	16.47	.15868	.02722	.49459	.08485	3.9	2.356	.962
164.69	96.96	.15503	.09128	.48322	.28451	16.2	1.661	1.441
96.96	164.69	.09128	.15503	.28451	.48322	22.7	-1.996	.957
16.47	96.02	.02722	.15868	.08485	.49459	-1.7	-8.366	-1.251
4.46	257.19	.00420	.24212	.01308	.75466	-63.0	-16.810	-5.799
4.46	257.19	.00420	.24212	.01308	.75466	-211.2	-26.356	-13.065
16.47	96.02	.02722	.15868	.08485	.49459	-500.6	-35.790	-23.116
96.96	164.69	.09128	.15503	.28451	.48322	-880.3	-43.784	-35.664
164.69	96.96	.15503	.09128	.48322	.28451	-1457.8	-49.024	-50.058
96.02	16.47	.15868	.02722	.49459	.08485	-2259.2	-50.361	-65.318
257.19	4.46	.24212	.00420	.75466	.01308	-3169.8	-46.935	-80.198
257.19	4.46	.24212	.00420	.75466	.01308	-2527.4	47.798	18.909
96.02	16.47	.15868	.02722	.49459	.08485	-2244.8	29.656	27.523
164.69	96.96	.15503	.09128	.48322	.28451	-1920.2	12.470	31.835
96.96	164.69	.09128	.15503	.28451	.48322	-1535.4	-2.385	32.319
16.47	96.02	.02722	.15868	.08485	.49459	-1166.6	-13.896	29.736
4.46	257.19	.00420	.24212	.01308	.75466	-814.6	-21.484	25.028
4.46	257.19	.00420	.24212	.01308	.75466	-532.4	-25.028	19.205
16.47	96.02	.02722	.15868	.08485	.49459	-337.5	-24.849	13.224
96.96	164.69	.09128	.15503	.28451	.48322	-171.4	-21.647	7.890
164.69	96.96	.15503	.09128	.48322	.28451	-81.4	-16.406	3.772
96.02	16.47	.15868	.02722	.49459	.08485	-41.4	-10.278	1.151
257.19	4.46	.24212	.00420	.75466	.01308	-11.2	-4.448	0.000
2543.16	2543.16	2.7141	2.7141	8.45964	8.45964			

*Calculations on pages 50 through 53

B5.2.1 Sample Problem (Cont'd)

27	28	29	30	31	32	33	34	35
$M_o \frac{S}{EI}$	$M_o x \frac{S}{EI}$	$M_o y \frac{S}{EI}$	$N_o \sin\psi \frac{S}{EA}$	$N_o \cos\psi \frac{S}{EA}$	$S_o \sin\psi \frac{S}{GA'}$	$S_o \cos\psi \frac{S}{GA'}$	Xy	Yx
							See Note III(5)	
24x1	24x13x1	24x14x1	25x16x2	25x17x2	26x16x3	26x17x3	Xx14	Yx13
10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}		
-1.35	-63.7	-8.4	-.06000	.00791	-.24357	.03206	32.403	-1193.318
.20	8.7	3.6	-.40466	.16761	-.51500	.21332	93.465	-1094.065
1.88	70.7	54.2	-.32457	.24905	-.87768	.67347	151.123	-954.908
2.64	76.2	99.4	.29928	-.39003	-.44727	.58289	196.948	-732.723
-.09	-1.6	-3.7	.59519	-1.4369	.27741	-.66972	225.649	-453.167
-7.31	-45.3	-344.1	.54037	-4.1050	.58105	-4.4140	246.120	-157.106
-24.52	152.0	-1154.2	-.84724	-6.4362	-1.3091	-9.9447	246.120	157.106
-25.81	461.4	-1113.9	-2.5462	-6.1472	-5.1259	-12.375	225.649	453.167
-102.19	2953.4	-3849.0	-6.5650	-8.5568	-16.668	-21.722	196.948	732.723
-169.22	6374.0	-4890.0	-9.5797	-7.3507	-30.489	-23.395	151.123	954.908
-116.48	5026.7	-2082.1	-8.6498	-3.5828	-34.968	-14.484	93.465	1094.065
-367.97	17320.4	-2280.2	-11.462	-1.5088	-61.044	-8.036	32.403	1193.318
-293.39	13810.1	1818.2	11.672	-1.5365	14.393	-1.895	-32.403	1193.318
-115.74	4994.7	2068.9	5.094	-2.1098	14.734	-6.103	-93.465	1094.065
-222.90	8395.8	6442.3	2.437	1.8698	19.390	-14.878	-151.123	954.908
-178.23	5151.2	6713.2	-.3576	.46605	15.105	-19.685	-196.948	732.723
-60.14	1075.1	2595.6	-.9886	2.3867	6.594	-15.919	-255.649	453.167
-94.56	586.0	4450.9	-.6906	5.2464	2.508	-19.051	-246.120	157.106
-61.80	-382.9	2908.8	.8046	6.1119	-1.924	-14.618	-246.120	-157.106
-17.40	-311.0	750.9	1.7679	4.2679	-2.932	-7.079	-225.649	-453.167
-19.89	-575.0	749.3	3.2458	4.2300	-3.688	-4.806	-196.948	-732.723
-9.45	-355.8	273.0	3.2059	2.4599	-2.267	-1.763	-151.123	-954.908
-2.13	-92.1	38.1	1.7654	.7312	-.6162	-.25523	-93.465	-1094.07
-1.30	-61.3	8.1	1.0862	.143	0.0000	0.00000	-32.403	-1193.32
-1887.15	64567.7	13248.0	-9.9626	-18.563	-89.533	-199.59		

B5.2.1 Sample Problem (Cont'd)

36	37	38	39	40	41	42
X sin ψ	X cos ψ	Y sin ψ	Y cos ψ	M	N	S
See Note III (5)				M_c + 24 − 34 + 35	25 + 37 + 38	26 + 36 − 39
X × 16	X × 17	Y × 16	Y × 17			
-5.184	.682	25.135	-3.309	-406.0	26.063	-1.555
-4.831	2.001	23.422	-9.702	-352.3	27.779	5.833
-4.148	3.183	20.113	-15.433	-258.4	24.957	12.726
-3.183	4.148	15.433	-20.113	-75.5	17.585	17.887
-2.001	4.831	9.702	-23.422	150.9	6.167	20.170
-.682	5.184	3.309	-25.135	365.2	-8.317	18.654
.682	5.184	-3.309	-25.135	531.2	-24.481	12.752
2.001	4.831	-9.702	-23.422	558.4	-40.661	2.307
3.183	4.148	-15.433	-20.113	486.9	-55.069	-12.368
4.148	3.183	-20.113	-15.433	177.4	-65.954	-30.477
4.831	2.001	-23.422	-9.702	-427.2	-71.782	-50.785
5.184	.682	-25.135	-3.309	-1177.5	-71.388	-71.705
5.184	-.682	-25.135	3.309	-470.3	21.981	20.784
4.831	-2.001	-23.422	9.702	-225.8	4.233	22.652
4.148	-3.183	-20.113	15.433	17.2	-10.826	20.550
3.183	-4.148	-15.433	20.113	225.6	-21.966	15.389
2.001	-4.831	-9.702	23.422	343.6	-28.429	8.315
.682	-5.184	-3.309	25.135	420.0	-29.997	.575
-.682	-5.184	3.309	25.135	388.0	-26.903	-6.612
-2.001	-4.831	9.702	23.422	266.3	-19.978	-12.199
-3.183	-4.148	15.433	20.113	124.2	-10.362	-15.406
-4.148	-3.183	20.113	15.433	-53.8	.524	-15.809
-4.831	-2.001	23.422	9.702	-210.6	11.143	-13.382
-5.184	-.682	25.135	3.309	-340.7	20.005	-8.493

B5.2.1 Sample Problem (Cont'd)

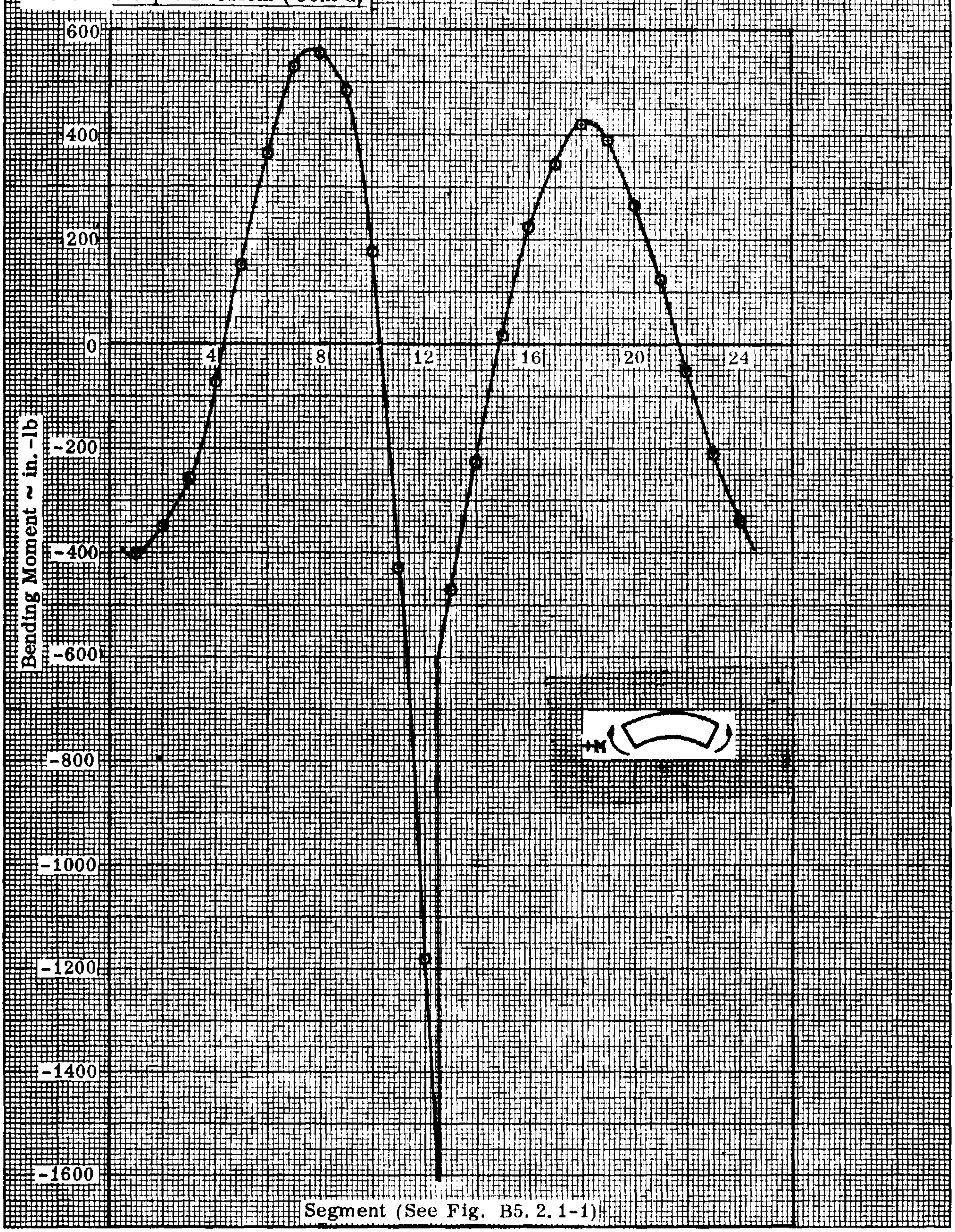


Figure B5.2.1-3 Bending Moment Diagram of Sample Problem

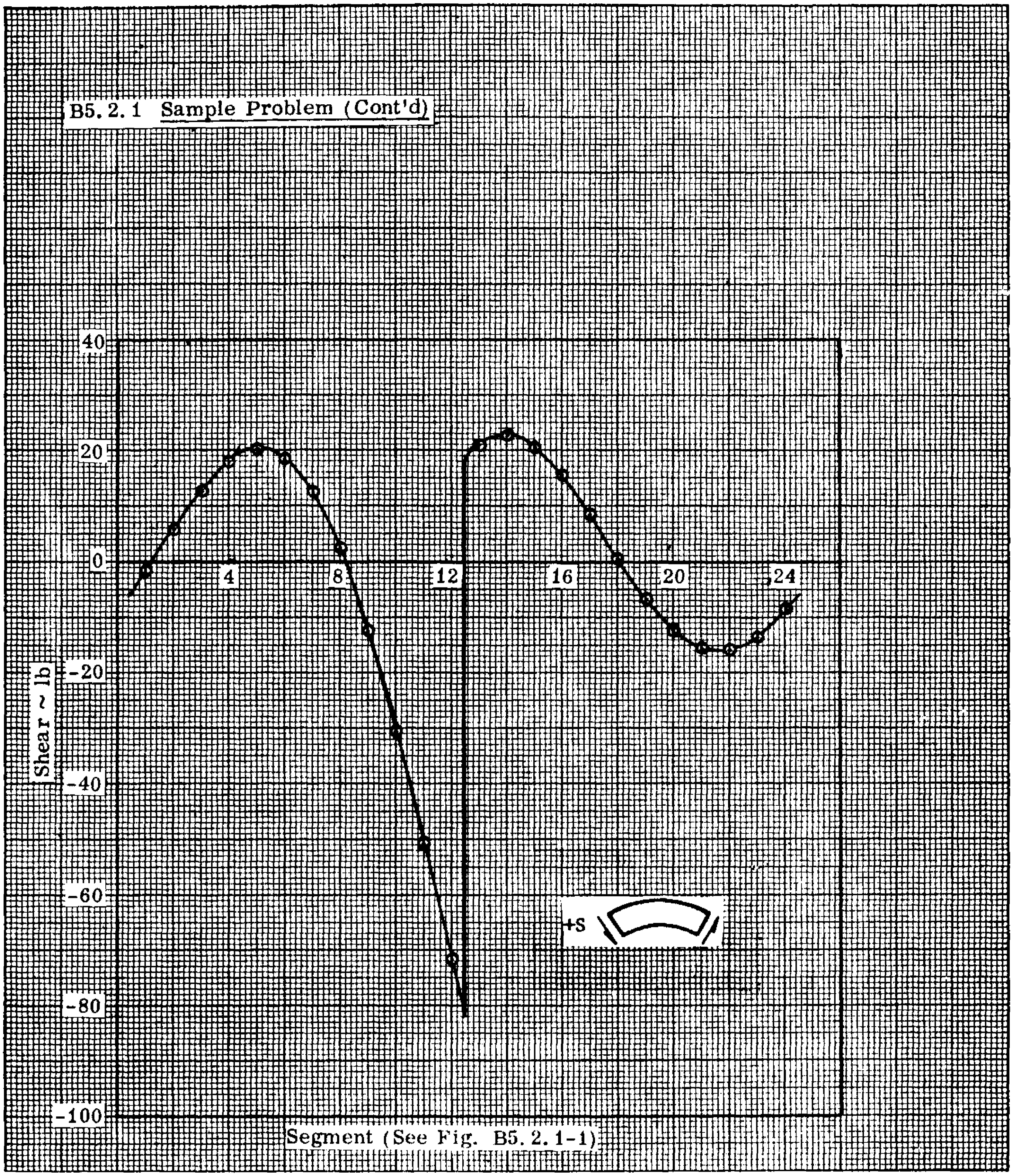


Figure B5.2.1-4 Shear Diagram of Sample Problem

B5. 2. 1 Sample Problem (Cont'd)

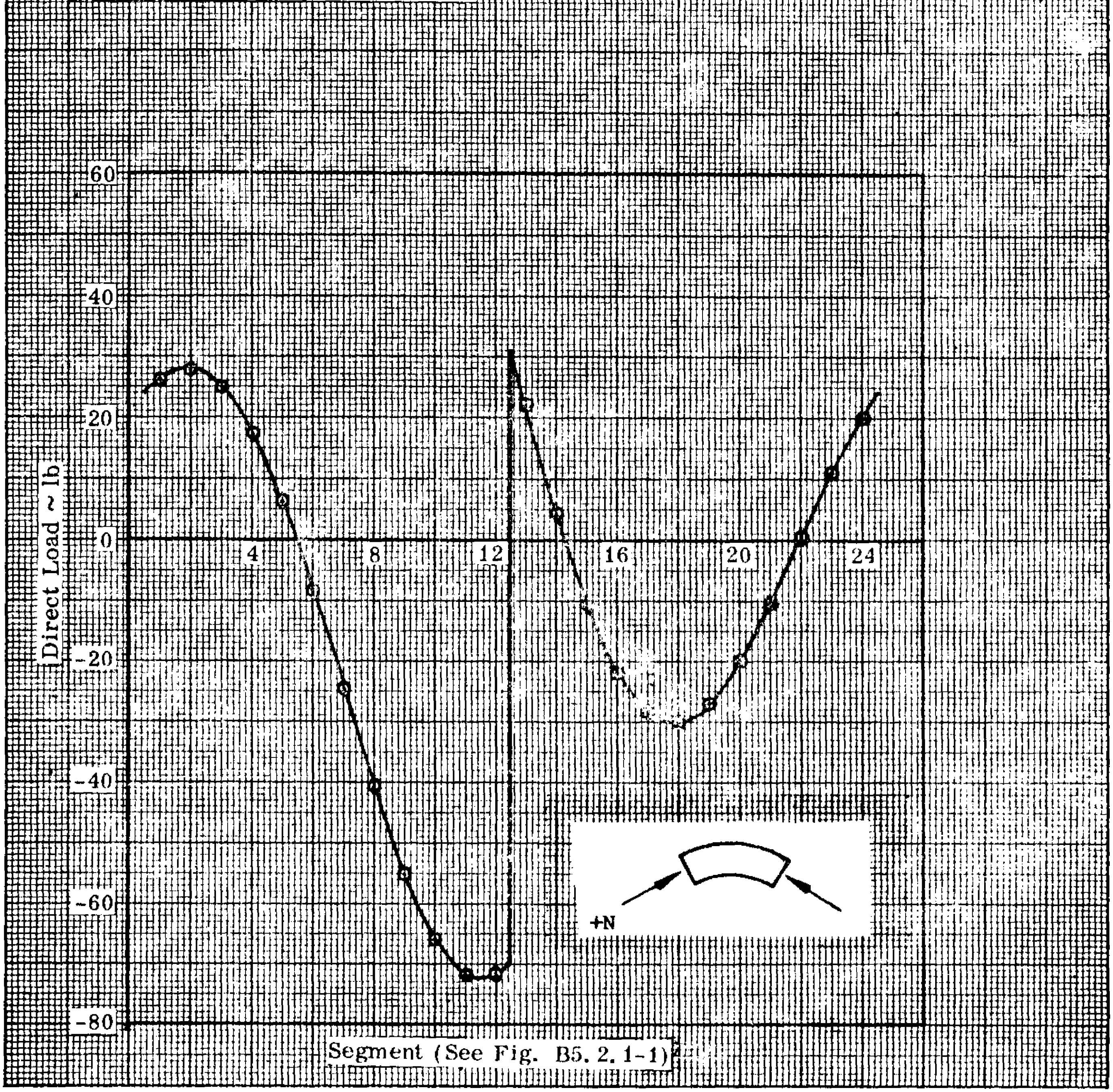


Figure B5. 2. 1-5 Direct Load Diagram of Sample Problem

B5.2.1 Sample Problem (Cont'd)

The values for M_o, N_o, and S_o are calculated and shown in the tables on pages 52 and 53. The necessary formulas for computing the shear flows are shown below. In these equations, counterclockwise shear flow is considered positive.

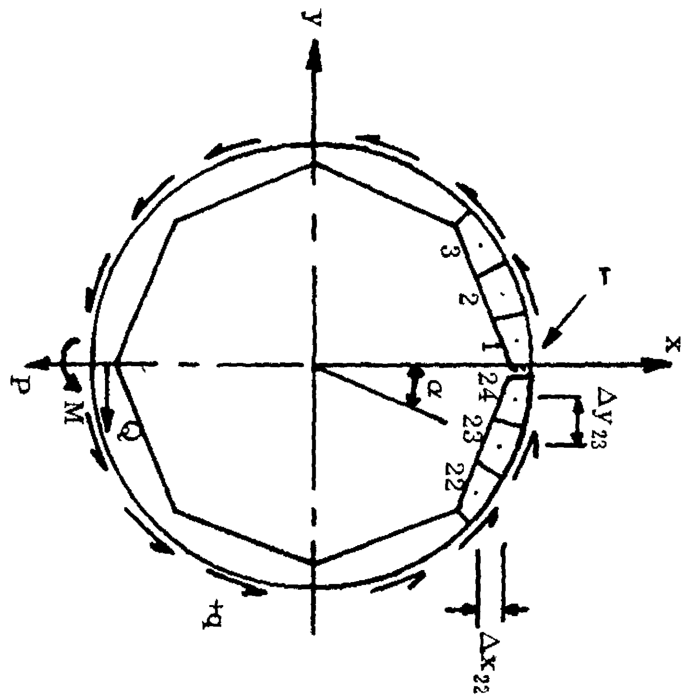


Figure B5.2.1-6 Symbols and Sign Conventions for Computation of M_o, N_o and S_o

$$q_{P_\alpha} = \frac{P}{\pi R_o} \sin \alpha \qquad q_{M_\alpha} = \frac{-M}{2\pi R_o^2}$$

$$q_{Q_\alpha} = \frac{Q}{\pi R_o} \sin (\alpha + 90°) - \frac{48.5\,Q}{2\pi R_o^2} \qquad q_\alpha = q_{P_\alpha} + q_{Q_\alpha} + q_{M_\alpha}$$

In order to perform the calculations for M_o, N_o, and S_o, the shear force acting on each segment must be known. This shear force is obtained by multiplying the average shear flow acting on the segment by the length over which it acts.

B5.2.1 Sample Problem (Concluded)

This average shear flow is determined by using Simpson's Rule. For the typical segment in Figure B5.2.1-7, this average shear flow is:

$$q_{avg.} = \frac{1}{6}\left(q'_\alpha + 4q''_\alpha + q'''_\alpha\right)$$

where:

q'_α = **Shear flow at left end of the segment.**

q''_α = **Shear flow at center of the segment.**

q'''_α = **Shear flow at right end of the segment.**

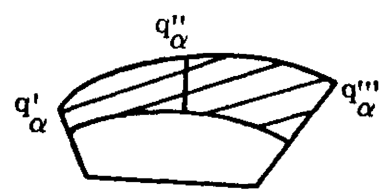


Figure B5.2.1-7 Shear Flow Acting on a Typical Segment

The formulas used for computing the values in the tables on pages 52 and 53 are listed under the tables. The index (n) refers to the column number and its range is indicated where necessary. The formulas for S_o, N_o, and M_o are applicable for all values of n, therefore, the index was omitted.

Seg.	α	$D = R_d$	h	Δx	Δy	$q_{avg.}$	F	H
	deg.							
24	7.5	47.477	5.046	.000	.000	.340	4.448	4.410
23	22.5	46.711	6.578	3.915	11.678	.457	5.982	9.937
22	37.5	47.477	5.046	5.489	11.027	.518	6.776	15.312
21	52.5	47.477	5.046	8.764	8.764	.518	6.776	19.437
20	67.5	46.711	6.578	11.027	5.489	.457	5.982	21.727
19	82.5	47.477	5.046	11.678	3.915	.340	4.448	22.307
18	97.5	47.477	5.046	12.394	.000	.174	2.279	22.010
17	112.5	46.711	6.578	11.678	-3.915	-.029	-.378	22.154
16	127.5	47.477	5.046	11.027	-5.489	-.255	-3.341	24.188
15	142.5	47.477	5.046	8.764	-8.764	-.490	-6.409	29.273
14	157.5	46.711	6.578	5.489	-11.027	-.716	-9.372	37.931
13	172.5	47.477	5.046	3.915	-11.678	-.919	-12.029	49.857
12	187.5	47.477	5.046	.000	-12.394	-1.085	-14.198	63.934
11	202.5	46.711	6.578	-3.915	-11.678	-1.202	-15.732	78.468
10	217.5	47.477	5.046	-5.489	-11.027	-1.262	-16.526	91.579
9	232.5	47.477	5.046	-8.764	-8.764	-1.262	-16.526	101.640
8	247.5	46.711	6.578	-11.027	-5.489	-1.202	-15.732	107.660
7	262.5	47.477	5.046	-11.678	-3.915	-1.085	-14.198	109.513
6	277.5	47.477	5.046	-12.394	.000	-.919	-12.029	107.943
5	292.5	46.711	6.578	-11.678	3.915	-.716	-9.372	104.357
4	307.5	47.477	5.046	-11.027	5.489	-.490	-6.409	100.455
3	322.5	47.477	5.046	-8.764	8.764	-.255	-3.341	97.805
2	337.5	46.711	6.578	-5.489	11.027	-.029	-.378	97.456
1	352.5	47.477	5.046	-3.915	11.678	.174	2.279	99.715

$1 \le n \le 23$

$$\Delta x_n = D_{n+1} \cos \alpha_{n+1} - D_n \cos \alpha_n$$

$$\Delta y_n = D_n \sin \alpha_n - D_{n+1} \sin \alpha_{n+1}$$

$$F_n = (q_{avg_n})(\Delta s_n)$$

$1 \le n \le 23$

$$H_n = H_{n+1} + F_n \cos \alpha_n$$

$$V_n = V_{n+1} + F_n \sin \alpha_n$$

$$M_n = H_{n+1} \Delta x_n - V_{n+1} \Delta y_n + M_{n+1} + F_n \frac{h}{2}$$

For n = 24:

$$\Delta x_n = \Delta y_n = 0, \quad F_n = (q_{avg_n})(\Delta s_n), \quad H_n = F_n \cos \alpha_n, \quad V_n = F_n \sin \alpha_n, \quad M_n = F_n \frac{h}{2}$$

Seg.	V	M	H'	V'	M'	S_o	N_o	M_o
24	.581	11.2	4.410	.581	11.2	.000	-4.448	-11.2
23	2.870	41.4	9.937	2.870	41.4	1.151	-10.278	-41.4
22	6.995	81.4	15.312	6.995	81.4	3.772	-16.406	-81.4
21	12.370	171.4	19.437	12.370	171.4	7.890	-21.647	-171.4
20	17.897	337.5	21.727	17.897	337.5	13.224	-24.849	-337.5
19	22.307	532.4	22.307	22.307	532.4	19.205	-25.028	-532.4
18	24.567	814.6	22.010	24.567	814.6	25.028	-21.484	-814.6
17	24.217	1166.6	22.154	24.217	1166.6	29.736	-13.896	-1166.6
16	21.567	1535.4	24.188	21.567	1535.4	32.319	-2.385	-1535.4
15	17.665	1920.2	29.273	17.665	1920.2	31.835	12.470	-1920.2
14	14.079	2244.8	37.931	14.079	2244.8	27.523	29.656	-2244.8
13	12.509	2527.4	49.857	12.508	2527.4	18.909	47.798	-2527.4
12	14.362	2646.6	-36.066	-85.638	3169.8	-80.198	-46.935	-3169.8
11	20.382	2512.3	-21.532	-79.618	2259.2	-65.318	-50.361	-2259.2
10	30.442	2264.6	-8.421	-69.558	1457.8	-50.058	-49.024	-1457.8
9	43.553	1687.1	1.640	-56.447	880.3	-35.664	-43.784	-880.3
8	58.088	753.7	7.660	-41.912	500.6	-23.116	-35.790	-500.6
7	72.164	-312.0	9.513	-27.836	211.2	-13.065	-26.356	-211.2
6	84.090	-1699.6	7.943	-15.910	63.0	-5.799	-16.810	-63.0
5	92.749	-3320.3	4.357	-7.251	1.7	-1.251	-8.366	-1.7
4	97.834	-4996.3	.455	-2.166	-22.7	.957	-1.996	22.7
3	99.868	-6742.5	-2.195	-.132	-16.2	1.441	1.661	16.2
2	100.012	-8381.9	-2.544	.012	-3.9	.962	2.356	3.9
1	99.715	-9925.7	-.285	-.285	11.7	.320	.246	-11.7

$\underline{13 \le n \le 24}$

$H'_n = H_n$

$V'_n = V_n$

$M'_n = M_n$

$\underline{1 \le n \le 12}$

$H'_n = H_n - Q$

$V'_n = V_n - P$

$$M'_n = M_n - P[D_n \sin(\alpha_n - 180°)] + Q[48.5 - D_n \cos(\alpha_n - 180°)] + M$$

$$S_o = -V' \cos\alpha + H' \sin\alpha, \quad N_o = -V' \sin\alpha - H' \cos\alpha, \quad M_o = -M'$$

B5.0.0 - FRAMES

References

1. Timoshenko, S., Theory of Structures, McGraw-Hill Book Company, Inc., New York, 1945.

2. Sutherland, H. and Bowman, H. L., Structural Theory, Fourth Edition, John Wiley & Sons, Inc., New York, 1954.

3. Wilbur, J. B. and Norris, C. H., Elementary Structural Analysis, First Edition, McGraw-Hill Book Co., Inc., New York, 1948.

4. Grinter, L. E., Theory of Modern Steel Structures, Vol. II, The Macmillan Co., New York, 1949.

5. Perry, D. J., Aircraft Structures, McGraw-Hill Book Co., Inc., New York, 1950.

6. Argyris, J. H., Dunne, P. C., Tye, W., et al., Structural Principles and Data, Fourth Edition, The New Era Publishing Co., Ltd., London, No date.

7. Roarke, R. J., Formulas for Stress and Strain, p 1120, Third Edition, McGraw-Hill Book Co., Inc., New York, 1954.