SECTION A3

COMBINED STRESSES

TABLE OF CONTENTS

A 3.0.0 Combined Stresses and Stress Ratio

A 3.1.0 Combined Stresses

When an element of structure is subjected to combined stresses such as tension, compression and shear, it is oftentimes necessary to determine resultant maximum stress values and their respective principal axes.

The solution may be attained through the use of equations or the graphical construction of Mohr's circle.

Relative Orientation and Equations of Combined Stresses

f_x and f_y are applied normal stresses.

f_s is applied shear stress.

f_{max} and f_{min} are the resulting principal normal stresses.

$f_{s_{max}}$ is the resulting principal shear stress.

θ is the angle of principal axes.

Sign Convention:

Tensile stress is positive.
Compressive stress is negative.
Shear stress is positive as shown.
Positive θ is counterclockwise as shown.

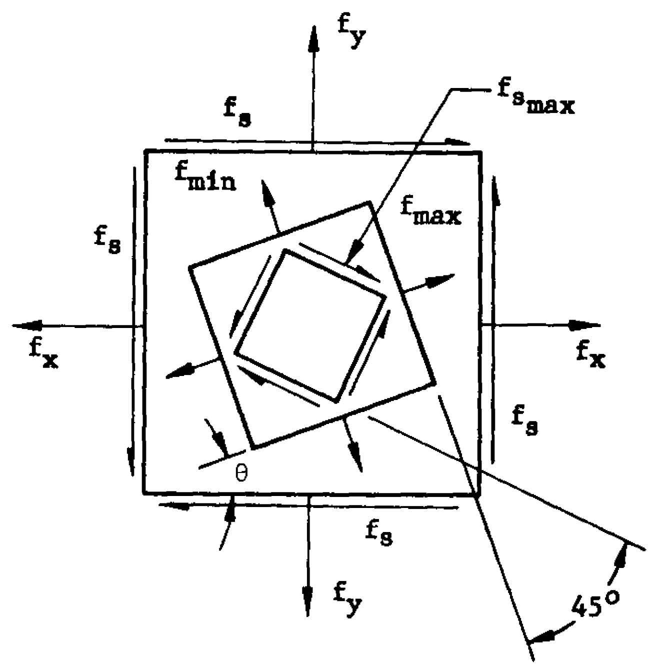


Fig. A 3.1.0-1

Note:
This convention of signs for shearing stress is adopted for this work only.

A 3.1.0 Combined Stresses (Cont'd)

Distributed Stresses on a 45° Element

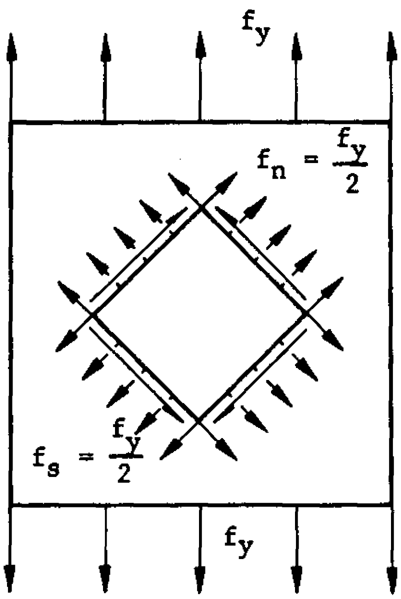


Fig. A 3.1.0-2
Pure Tension

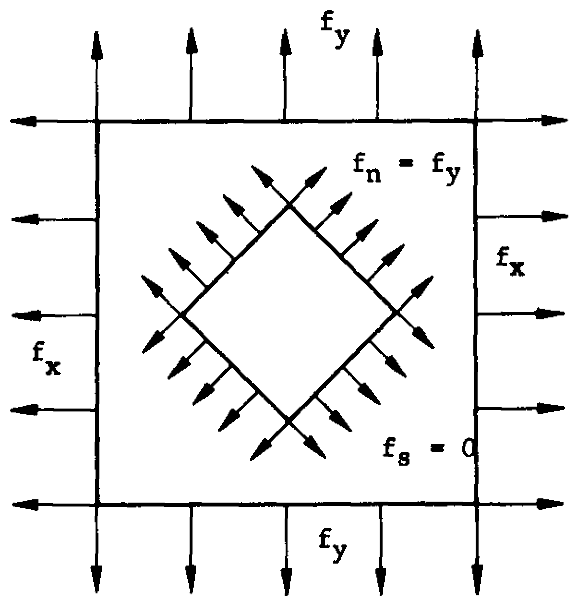


Fig. A 3.1.0-3
Equal Biaxial Tension

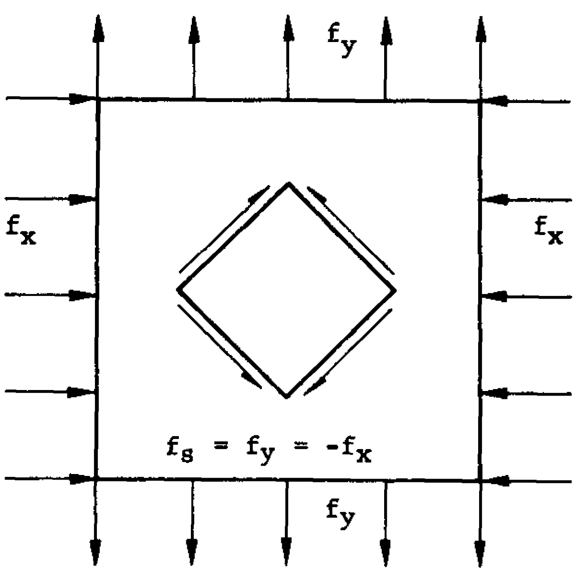


Fig. A 3.1.0-4
Equal Tension &
Compression

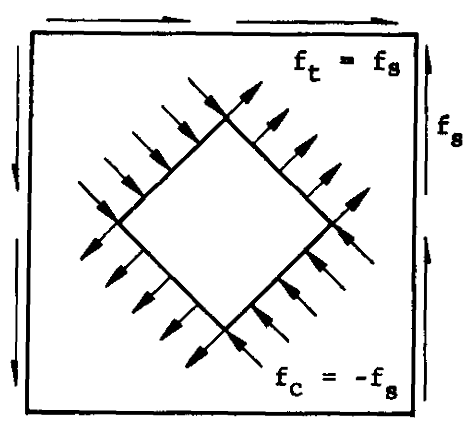


Fig. A 3.1.0-5
Pure Shear

A 3.1.0 Combined Stresses (Cont'd)

$$f_{max} = \frac{f_x + f_y}{2} + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + f_s^2} \quad \ldots\ldots\ldots\ldots (1)$$

$$f_{min} = \frac{f_x + f_y}{2} - \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + f_s^2} \quad \ldots\ldots\ldots\ldots (2)$$

$$\text{TAN } 2\theta = \frac{2f_s}{f_x - f_y}$$ [The solution results in two angles representing the principal axes of f_{max} and f_{min}.] .... (3)

$$f_{s_{max}} = \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + f_s^2} \quad \text{(Disregard Sign)} \ldots\ldots\ldots\ldots (4)$$

Constructing Mohr's Circle (for the stress condition shown in Fig. A 3.1.0-6a)

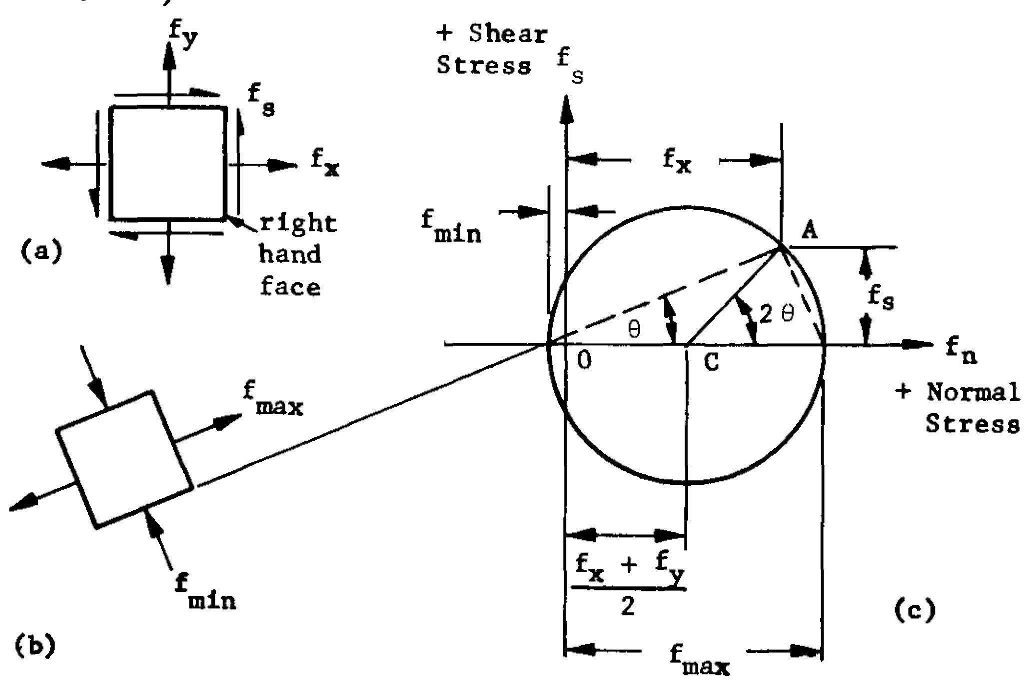


Fig. A 3.1.0-6

A 3.1.0 Combined Stresses (Cont'd)

1. Make a sketch of an element for which the normal and shearing stresses are known and indicate on it the proper sense of these stresses.

2. Set up a rectangular co-ordinate system of axes where the horizontal axis is the normal stress axis, and the vertical axis is the shearing stress axis. Directions of positive axes are taken as usual, upward and to the right.

3. Locate the center of the circle, which is on the horizontal axis at a distance of $(f_x + f_y)/2$ from the origin. Tensile stresses are positive, compressive stresses are negative.

4. From the right-hand face of the element prepared in step (1), read off the values for f_x and f_s and plot the controlling point "A". The co-ordinate distances to this point are measured from the origin. The sign of f_x is positive if tensile, negative if compressive; that of f_s is positive if upward, negative if downward.

5. Draw the circle with center found in step (3) through controlling point "A" found in step (4). The two points of intersection of the circle with the normal-stress axis give the magnitudes and sign of the two principal stresses. If an intercept is found to be positive, the principal stress is tensile, and conversely.

6. To find the direction of the principal stresses, connect point "A" located in step (4) with the intercepts found in step (5). The principal stress given by the particular intercept found in step (5) acts normal to the line connecting this intercept point with the point "A" found in step (4).

7. The solution of the problem may then be reached by orienting an element with the sides parallel to the lines found in step (6) and by indicating the principal stresses on this element.

To determine the maximum or the principal shearing stress and the associated normal stress:

1. Determine the principal stresses and the planes on which they act per previous procedure.

2. Prepare a sketch of an element with its corners located on the principal axes. The diagonals of this element will thus coincide with the directions of the principal stresses. (See Fig. A 3.1.0-7).

3. The magnitude of the maximum (principal) shearing stresses acting on mutually perpendicular planes is equal to the radius of the circle. These shearing stresses act along the faces of the element prepared in step (2) toward the diagonal, which coincides with the direction of the algebraically greater normal stress.

A 3.1.0 Combined Stresses (Cont'd)

4. The normal stresses acting on all faces of the element are equal to the average of the principal stresses, considered algebraically. The magnitude and sign of these stresses are also given by the distance from the origin of the co-ordinate system to the center of Mohr's circle.

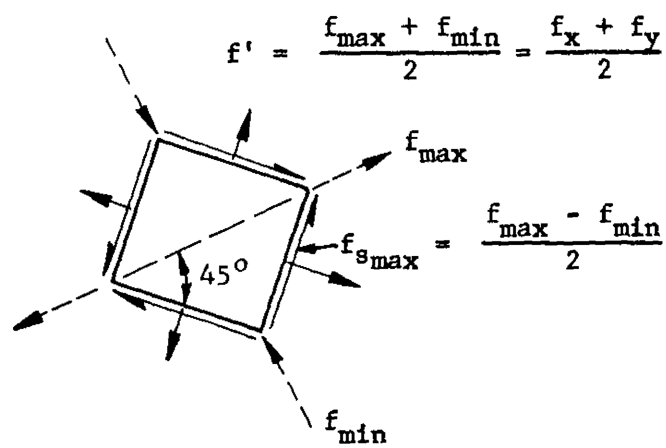


Fig. A 3.1.0-7

A 3.1.0 Combined Stresses (Cont'd)

Mohr's Circle for Various Loading Conditions

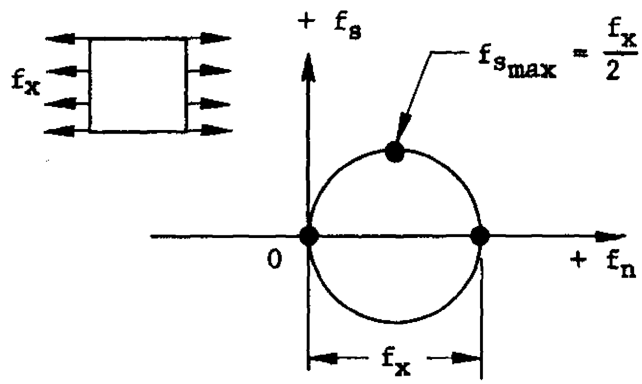


Fig. A 3.1.0-8 Simple Tension

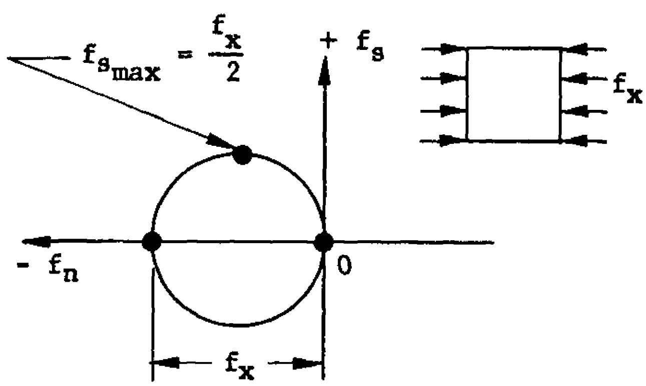


Fig. A 3.1.0-9 Simple Compression

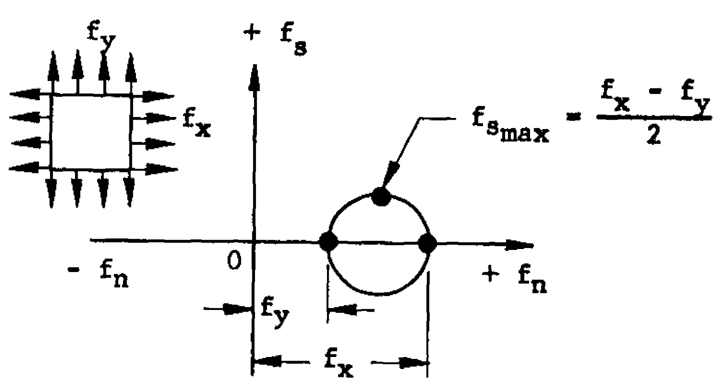


Fig. A 3.1.0-10 Biaxial Tension

A 3.1.0 Combined Stresses (Cont'd)

Mohr's Circle for Various Loading Conditions

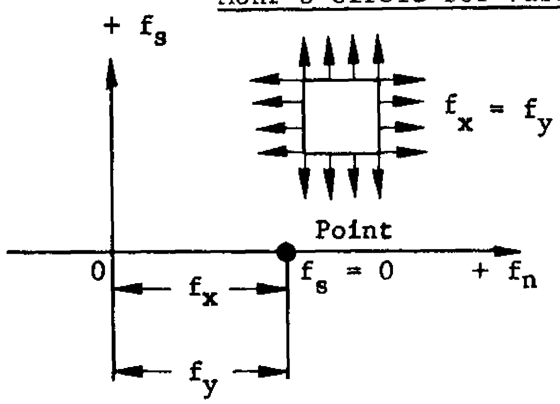


Fig. A 3.1.0-11 Equal Biaxial Tension

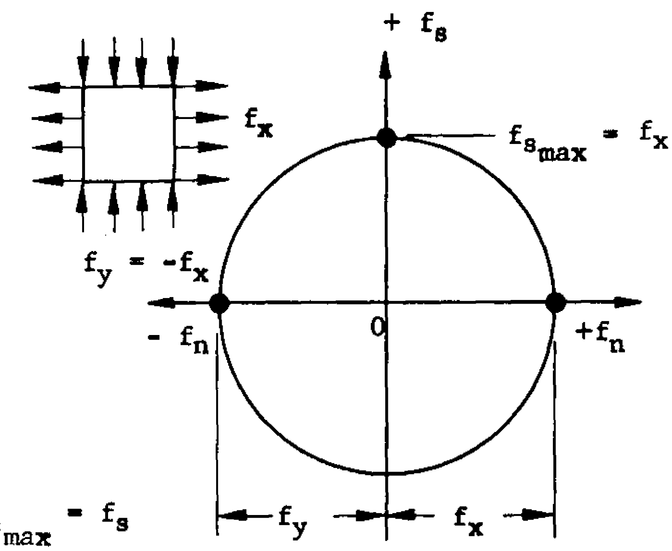


Fig. A 3.1.0-12
Equal Tension and Compression

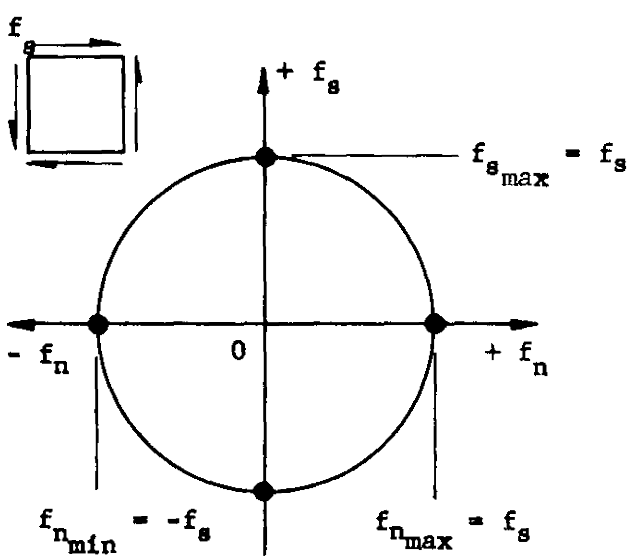


Fig. A 3.1.0-13
Pure Shear

A 3.2.0 Stress Ratios, Interaction Curves, and Factor of Safety

A means of predicting structural failure under combined loading without determining principal stresses is known as the interaction method.

The basis for this method is as follows:

1. The strength under each simple loading condition (tension, shear, bending, buckling, etc.) is determined by test or theory.
2. The combined loading condition is represented by either load or stress ratios, "R" where

$$R = \frac{\text{APPLIED LOAD OR STRESS}}{\text{FAILING LOAD OR STRESS}}$$

Failing can mean yield, rupture, buckling, etc.

The effect of one loading R_1 on another simultaneous loading R_2 is represented by an equation or interaction curve involving R_1 and R_2. The equation or curve may have been determined by theory, by test, or by a combination of both.

A schematic interaction curve is shown in Fig. A 3.2.0-1. Type of material or size effects will not influence it. This curve represents all the possible combinations of R_1 and R_2 that will cause failure.

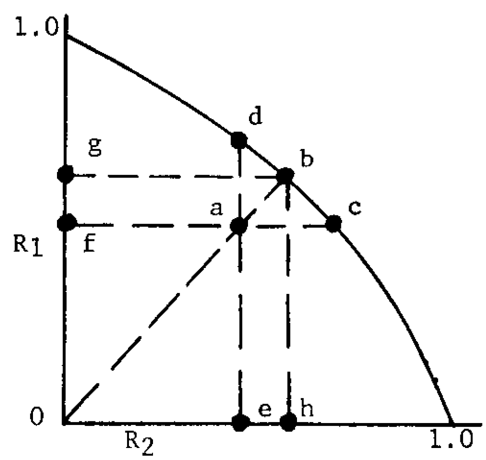


Fig. A 3.2.0-1

Using the curve:

1. Let the value of R_1 and R_2 locate point a.
2. R_1 and R_2 can increase proportionately until failure occurs at point b.
3. If R_1 remains constant, R_2 can increase until failure occurs at point c.
4. If R_2 remains constant, R_1 can increase until failure occurs at point d.
5. The factor of safety for (2) is F. S. = (ob ÷ oa), or (oh ÷ oe), (or og ÷ of) and the factor of safety for (3) is F. S. = (fc ÷ fa).

A 3.2.0 Stress Ratios, Interaction Curves, and Factor of Safety (Cont'd)

In general, the formula for the factor of safety stated analytically for interaction equations where the exponents are only 1 or 2 (one term may be missing) is as follows:

$$F.S. = \frac{2}{\left[R' + \sqrt{(R')^2 + (R'')^2}\right]} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where

R' designates the sum of all first-power ratios.

R'' designates the sum of all second-power ratios.

A 3.2.1 <u>A Theoretical Approach to Interaction</u>

For combining normal and shear stresses, the principal stress equations are convenient to use.

Let F and F_s be defined as the failing stress, such as yielding or rupture.

Let $k = F_s/F$; tests of most materials will show this ratio to vary from 0.50 to 0.75.

$$R_f = f/F; \quad R_s = f_s/F_s$$

Maximum Normal Stress Theory

$$f_{max} = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2} \qquad \text{Ref Eq. (1) Sec. A 3.1.0}$$

Divide by F; replace f_s by $R_s F_s$, f/F by R_f and F_s/F by k.

The resulting equation when $f_{max} = F$ is

$$1 = \frac{R_f}{2} + \sqrt{\left(\frac{R_f}{2}\right)^2 + (kR_s)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

A plot of this equation for k = 0.50 and k = 0.70 is shown in Fig. A 3.2.1-1.

Maximum Shear Stress Theory

$$f_{s_{max}} = \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2} \qquad \text{Ref Eq. (4) Sec. A 3.1.0}$$

Divide by F_s; replace f by $R_f F$, f_s/F_s by R_s and F/F_s by 1/k.

The resulting equation when $f_{s_{max}} = F_s$ is

$$1 = \sqrt{\left(\frac{R_f}{2k}\right)^2 + R_s^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

A plot of this equation for k = 0.50 and k = 0.70 is shown in Fig. A 3.2.1-1.

A 3.2.1 <u>A Theoretical Approach to Interaction (Cont'd)</u>

Conclusion

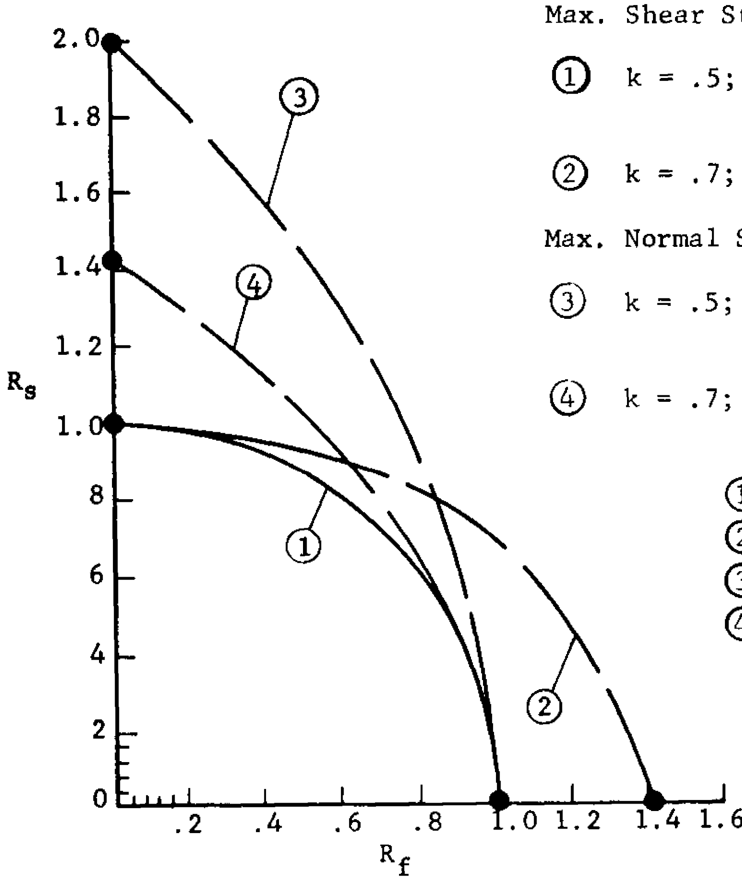
From the foregoing analysis, only Equation (2) with k = 0.5 is valid for all values of R_f and R_s. It is conservatively safe to use the resulting Equation (3) for values of k ranging from 0.5 to 0.7, since all values within curve ① must also be within the other curves. The use of other curves of Fig. A 3.2.1-1 may lead to unconservative results.

$$R_f^2 + R_s^2 = 1 \quad \ldots\ldots\ldots\ldots \quad (3)$$

and the Factor of Safety

$$F.S. = \frac{1}{\sqrt{R_f^2 + R_s^2}} \quad \ldots\ldots\ldots\ldots \quad (4)$$

For the graphical solution for Factor of Safety, the curve $R_1^2 + R_2^2 = 1$ of Fig. A 3.4.0-1 may be used.



Max. Shear Stress Theory

① k = .5; $R_f{}^2 + R_s{}^2 = 1$

② k = .7; $.5R_f{}^2 + R_s{}^2 = 1$

Max. Normal Stress Theory

③ k = .5; $R_f + \sqrt{R_f{}^2 + R_s{}^2} = 2$

④ k = .7; $R_f + \sqrt{R_f{}^2 + (1.4\ R_s)^2} = 2$

① Valid
② Partly Valid
③ Invalid
④ Partly Valid

Fig. A 3.2.1-1

A 3.3.0 Interaction for Beam-Columns

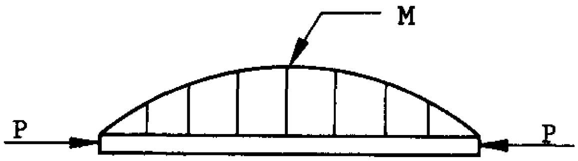


Fig. A 3.3.0-1
Sinusoidal Moment Curve

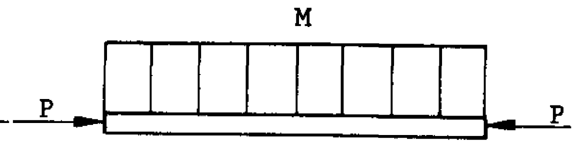


Fig. A 3.3.0-2
Constant Moment Curve

P = applied load.

$P_e = \frac{\pi^2 EI}{L^2}$ (Euler load). (Reference Section C 1.0.0).. (1)

P_o = buckling load $= \frac{\pi^2 E_t I}{L^2}$ .... (2)

or applicable short column formula. (Reference Section C 1.0.0)

M = maximum applied bending moment as a beam only.

M_o = ultimate bending moment as a beam only. (Reference Section B 4.0.0)

$R_a = \frac{P}{P_o}$ (column stress ratio) .... (3)

$R_b = \frac{M}{M_o}$ (beam stress ratio) .... (4)

$$f = \frac{P}{A} + k\,\frac{Mc}{I}$$

from which the interaction equation is:

$R_a + kR_b = 1$ .... (5)

Let $\eta = \frac{P_o}{P_e} = \frac{E_t}{E}$ (plasticity coefficient)............... (6)

For sinusoidal bending moment curves

$$k = \frac{1}{1 - P/P_e}$$

$R_b = (1 - R_a)\,(1 - \eta\,R_a)$ .... (7)

A 3.3.0 Interaction for Beam-Columns (Cont'd)

Interaction curves for various values of η are shown in Fig. A 3.3.1-5.

For constant bending moment curves

$$k = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{P/P_e}\right)}$$

$$R_b = (1 - R_a)\cos\left(\frac{\pi}{2}\sqrt{\eta\, R_a}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Interaction curves for various values of η are shown in Fig. A 3.3.1-6.

Conclusion

Comparison of Figs. A 3.3.1-5 and A 3.3.1-6 show that significant changes in shape of the primary bending moment diagram do not greatly influence the interaction curves. Therefore, Figs. A 3.3.1-5 and A 3.3.1-6 should be adequate for many types of simple beam columns.

A 3.3.1 Interaction for Eccentrically Loaded and Crooked Columns

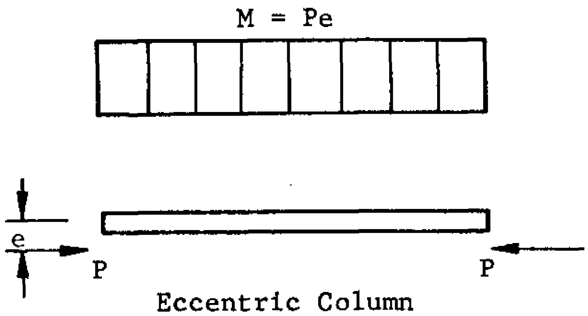


Fig. A 3.3.1-1

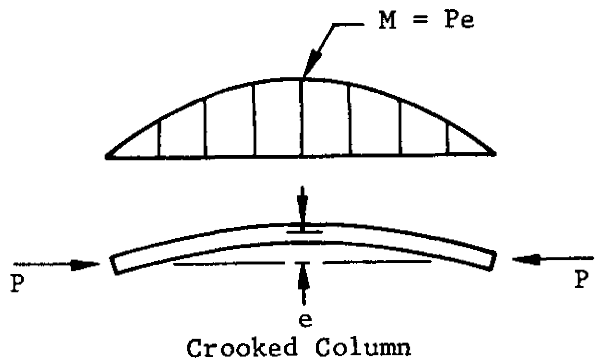


Fig. A 3.3.1-2

Reference Section A 3.3.0 for beam-column terms

$R_e = \frac{e}{e_o}$ (eccentricity ratio) .... (1)

$e_o = \frac{M_o}{P_o}$ (base eccentricity, which is that required for P_o to induce a moment M_o) ... (2)

For a particular e, M would be a linear function of P as shown in Fig. A 3.3.1-3. A family of such lines could be drawn which would represent all eccentric columns.

To obtain Fig. A 3.3.1-4 (a nondimensional one-one diagram of the same form as the interaction curves of Figs. A 3.3.1-5 and A 3.3.1-6), P, M, and e of Fig. A 3.3.1-3 may be divided by P_o, M_o and e_o respectively.

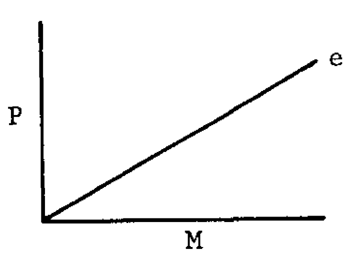


Fig. A 3.3.1-3

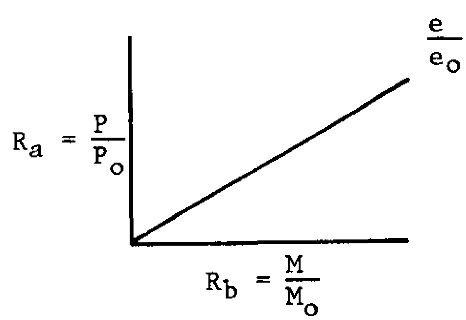


Fig. A 3.3.1-4

A 3.3.1 Interaction for Eccentrically Loaded and Crooked Columns (Cont'd)

In using Fig. A 3.3.1-6 for eccentric columns and Fig. A 3.3.1-5 for crooked columns the following steps are taken:

1. Determine P_o, the buckling load by $\pi^2 E_t I/L^2$ or applicable short column formula.

2. Calculate $P_e = \pi^2 EI/L^2$, the Euler load.

3. Determine M_o, the ultimate bending moment as a beam only using Section B 4.0.0.

4. Calculate $e_o = M_o/P_o$, the base eccentricity.

5. Calculate $R_e = e/e_o$.

6. Calculate $\eta = P_o/P_e$, the plasticity coefficient.

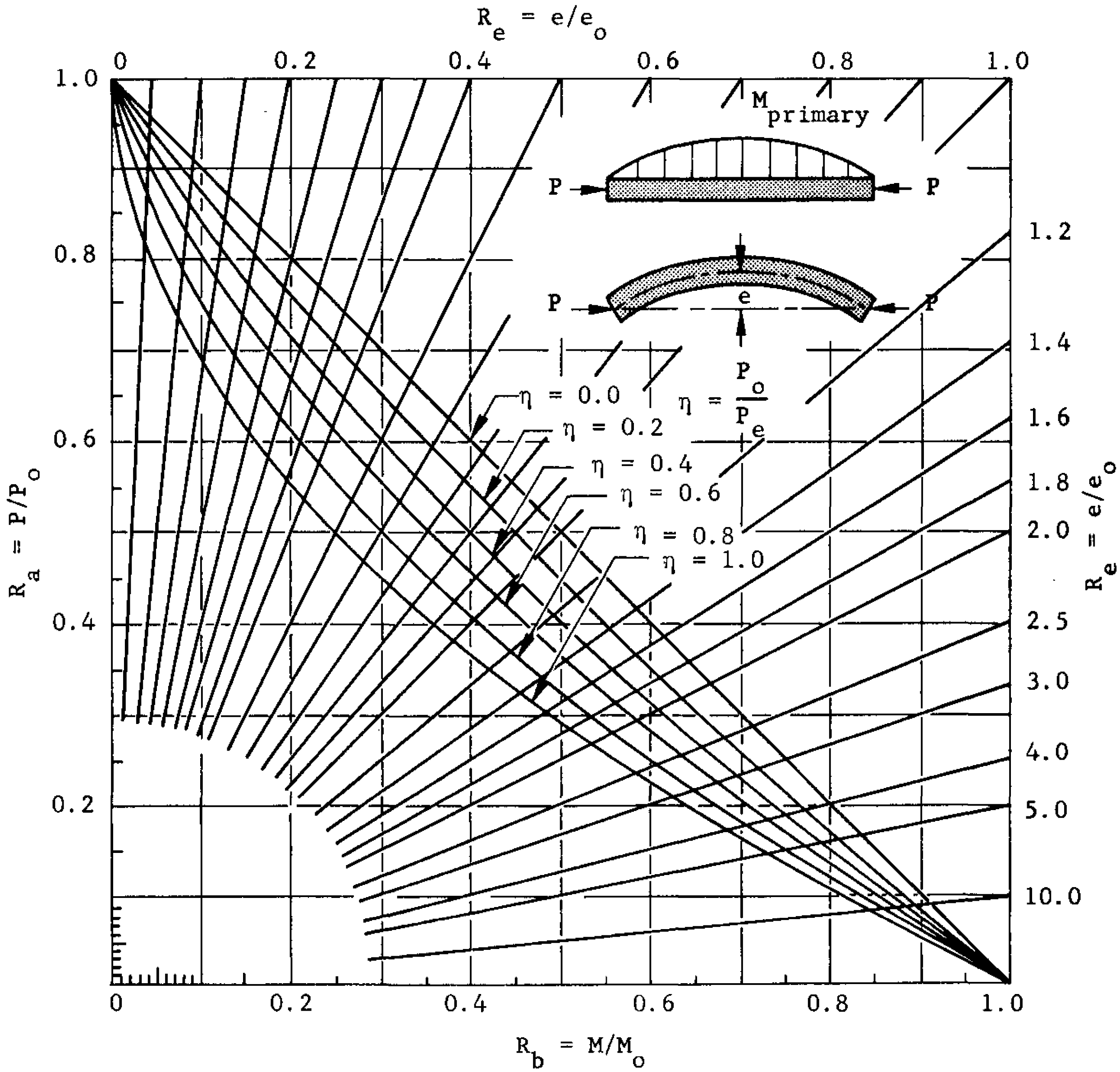
7. Knowing R_e and η, $R_a = P/P_o$ may be determined from the appropriate curve. This value of R_a corresponds to a Factor of Safety of 1.0.

8. The ultimate load is $P_u = P_o \times R_a$.

9. The Factor of Safety for an applied load P is

$$F.S. = \frac{P_u}{P}$$

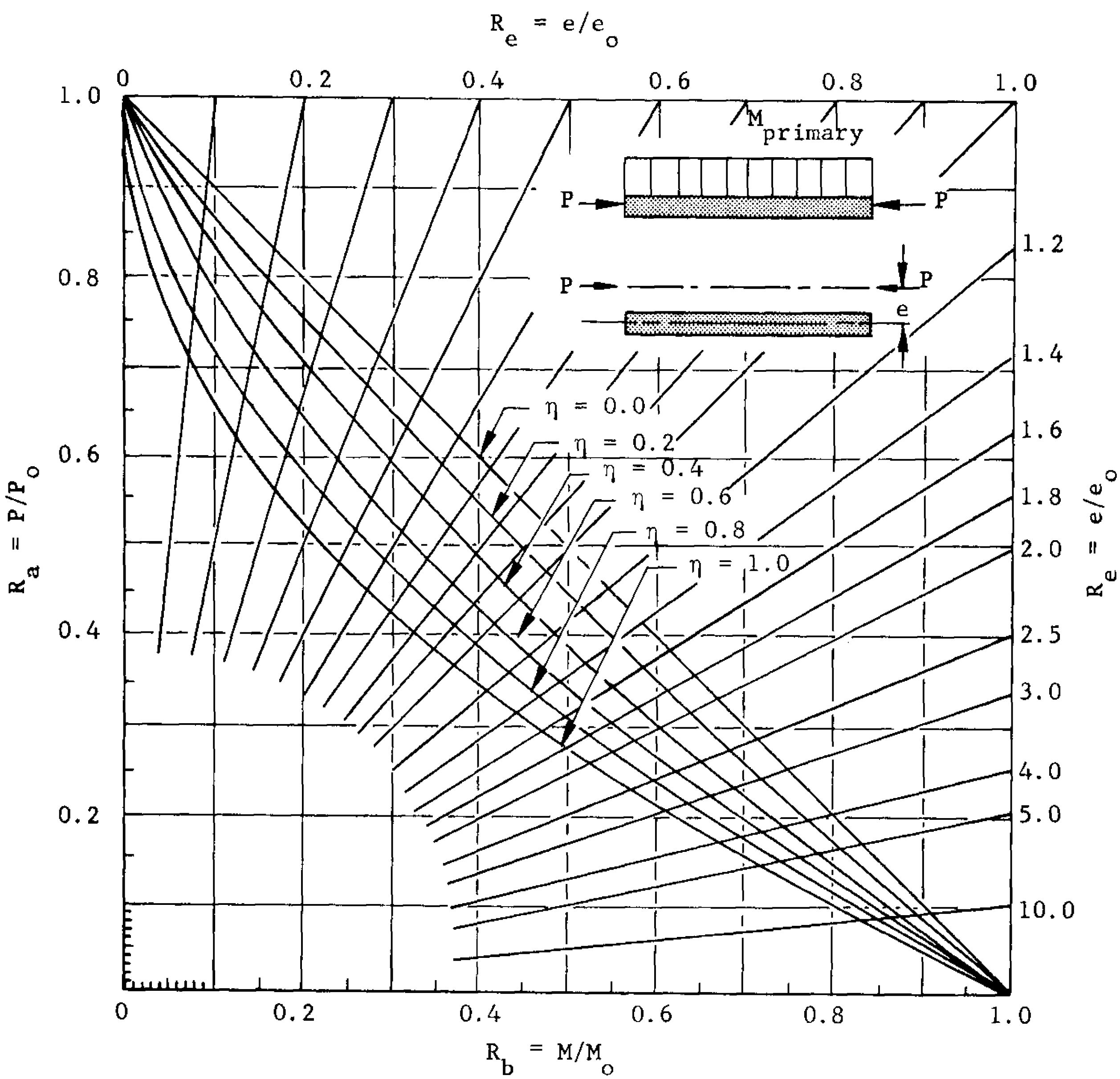
A 3.3.1 Interaction for Eccentrically Loaded and Crooked Columns (Cont'd)



Interaction Curves for Straight or Crooked Columns
with Sinusoidal Primary Bending Moment and Compression

Fig. A 3.3.1-5

A 3.3.1 Interaction for Eccentrically Loaded and Crooked Columns (Cont'd)



Interaction Curves for Columns with Constant Primary Bending Moment and Axial or Eccentric Compression

Fig. A 3.3.1-6

Table A 3.4.0-1

STRUCTURE	LOADING COMBINATION	FIGURE	INTERACTION EQUATION	EQ. FOR FACTOR OF SAFETY	REMARKS
Compact	Biaxial Tension or Biaxial Compression		$R_x = \frac{f_x}{F}$; $R = \frac{f_y}{F}$	$\frac{1}{R_{max}}$	Use R_x or R_y, whichever is greater
Compact and Round Tubes (a)	Axial and Bending Stresses	A 3.4.0-1	$R_s + R_b = 1$	$\frac{1}{R_a + R_b}$	For R_b, see Sect. B 4.0.0 on Plastic Bending
	Normal and Shear Stresses	A 3.4.0-1	$R_f^2 + R_s^2 = 1$ $R_f = R_a + R_b$	$\frac{1}{\sqrt{R_f^2 + R_s^2}}$	(b) for $.5 < \frac{F_s}{F} < .75$; For all other values use max. stress equations or Mohr's circle
Round Tubes	Bending, Torsion and Compression		$R_b^2 + R_{st}^2 = (1 - R_c)^2$	$\frac{1}{R_c + \sqrt{R_b^2 + R_{st}^2}}$	
Stream-Line Tubes	Bending and Torsion	A 3.4.0-1	$R_b + R_{st} = 1$	$\frac{1}{R_b + R_{st}}$	
Bolts	Tension and Shear	A 3.4.0-1 A 3.4.0-2	$R_t^2 + R_s^3 = 1$		

A 3.4.0 General Interaction Relationships (Cont'd)

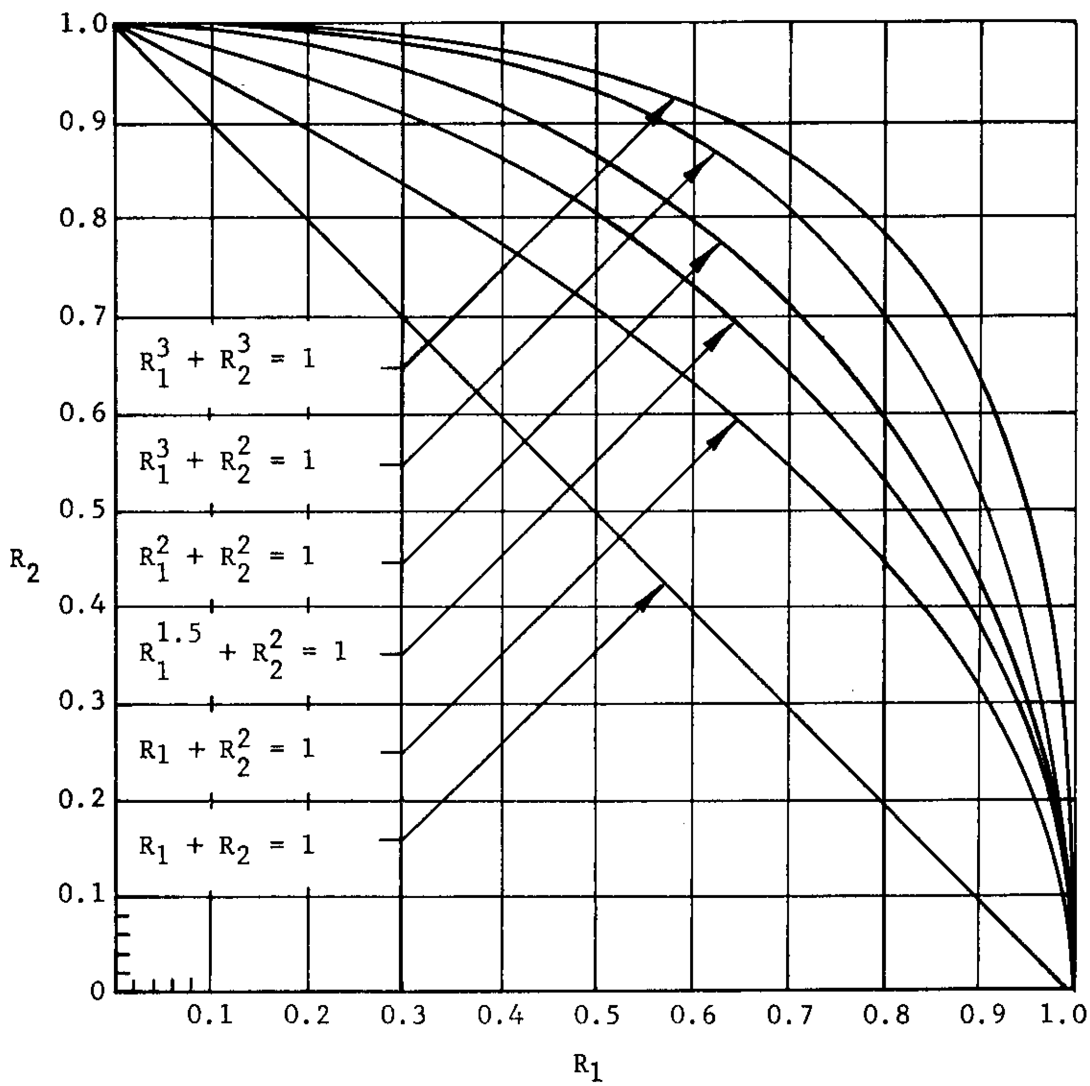
Table A 3.4.0-1 (Cont'd)

NOTE: Care must be exercised in determining whether to check Factor of Safety for limit or ultimate loads.

(a) For round tubes in compression see Section C.3.0.0.

(b) See Section A 3.2.1 for discussion of range.

A 3.4.0 General Interaction Relationships (Cont'd)



Interaction Curves

Fig. A 3.4.0-1

A 3.4.0 General Interaction Relationships (Cont'd)

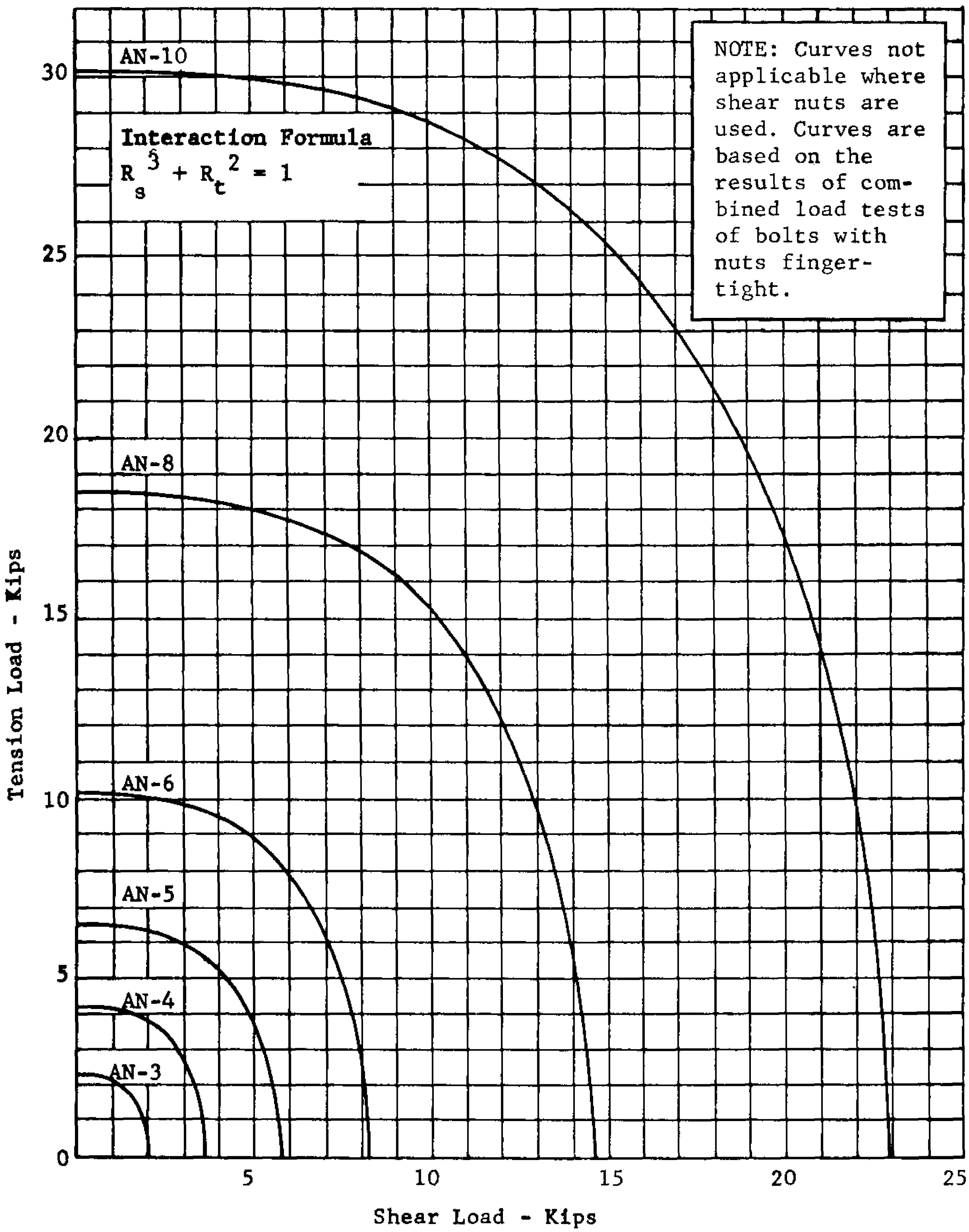
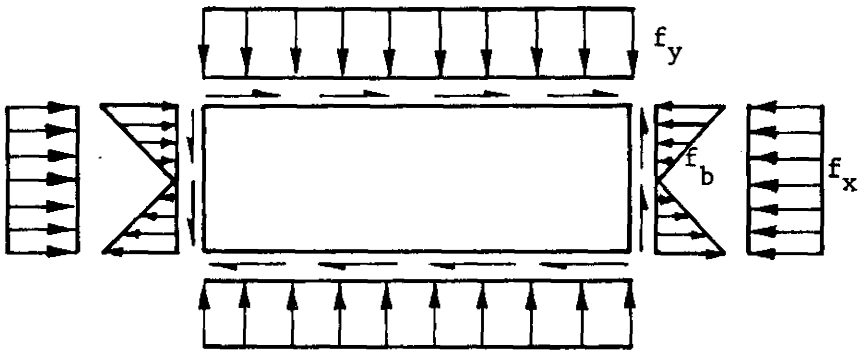


Fig. A 3.4.0-2

AN Steel Bolts (125,000 H.T.) Interaction Curves*

A 3.5.0 Buckling of Rectangular Flat Plates Under Combined Loading

NOTE: See discussion in Sec. C 2.1.4



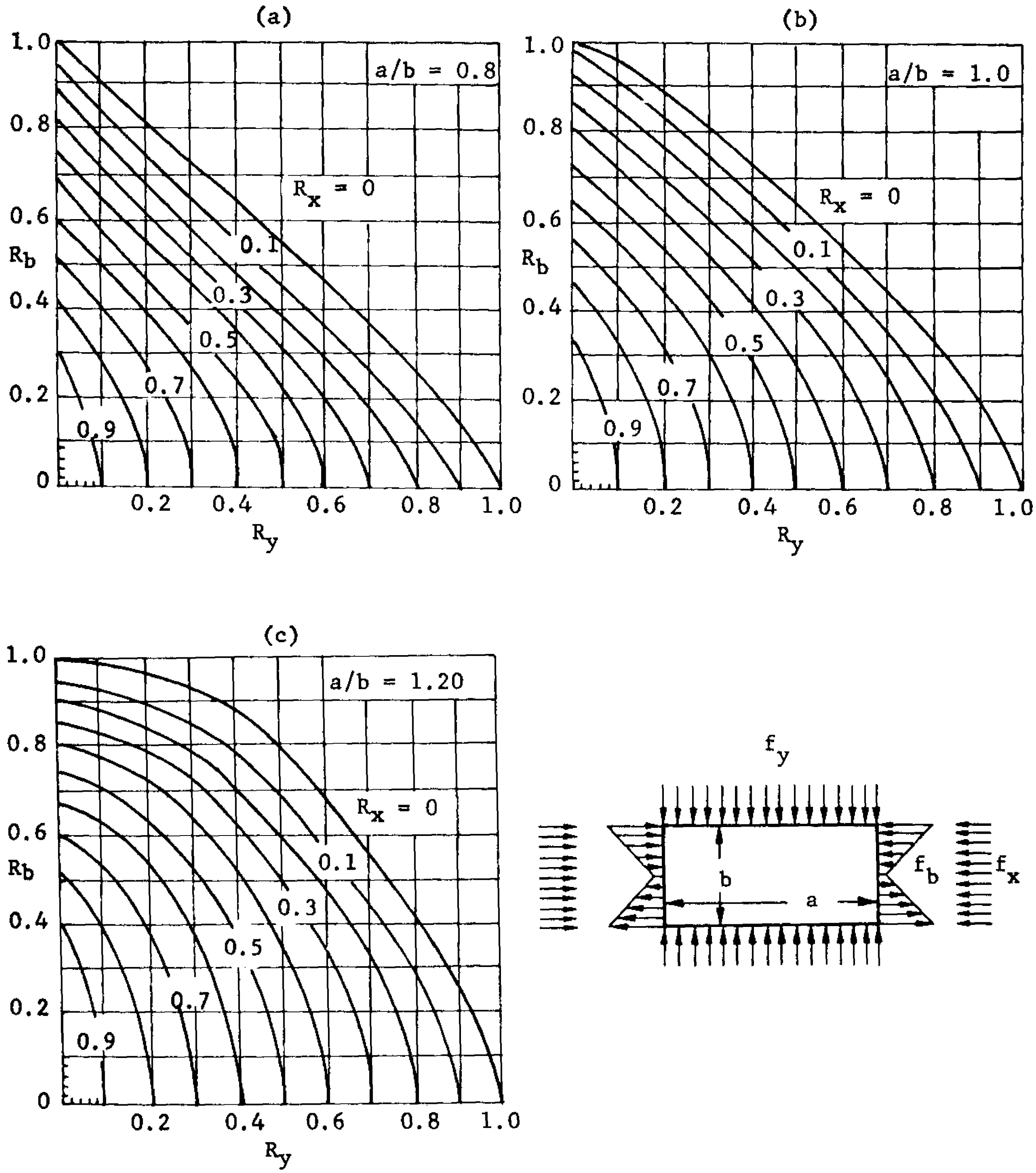
Combined Loading

Fig. A 3.5.0-1

Table A 3.5.0-1

THEORY	LOADING COMBINATION	FIGURE	INTERACTION EQUATION	EQ FOR FACTOR OF SAFETY
Elastic	Biaxial Compression	A 3.5.0-2	For plates that buckle in sq. waves, $R_x + R_y = 1$	$\frac{1}{R_x + R_y}$
	Longitudinal Compression and Shear		For Long Plates, $R_c + R_s^2 = 1$	$\frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}}$
	Longitudinal Compression and Bending	A 3.5.0-2		
	Bending and Shear	A 3.4.0-1	$R_b^2 + R_s^2 = 1$	$\frac{1}{\sqrt{R_b^2 + R_s^2}}$
	Bending, Shear, & Transverse Compression	A 3.5.0-3		
	Longitudinal Compression, Bending and Transverse Compression	A 3.5.0-2		
Inelastic	Longitudinal Compression and Shear	A 3.4.0-1	$R_c^2 + R_s^2 = 1$	$\frac{1}{\sqrt{R_c^2 + R_s^2}}$

A 3.5.0 Buckling of Rectangular Flat Plates Under Combined Loading (Cont'd)



Interaction Curves for Simply Supported Flat Rectangular Plates Under Combined Biaxial-Compression and Longitudinal Bending Loadings

Fig. A 3.5.0-2

A 3.5.0 Buckling of Rectangular Flat Plates under Combined Loading (Cont'd)

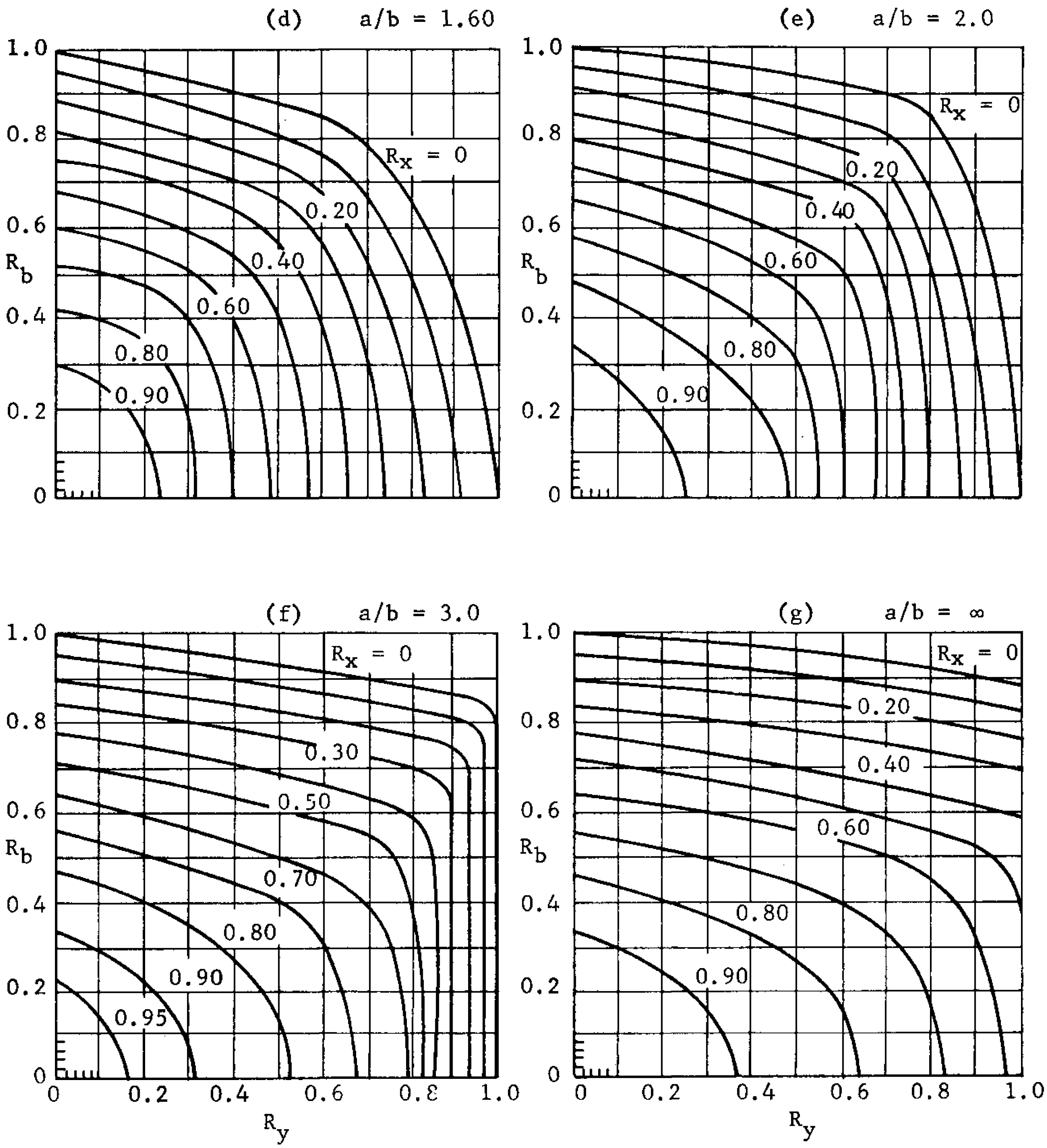
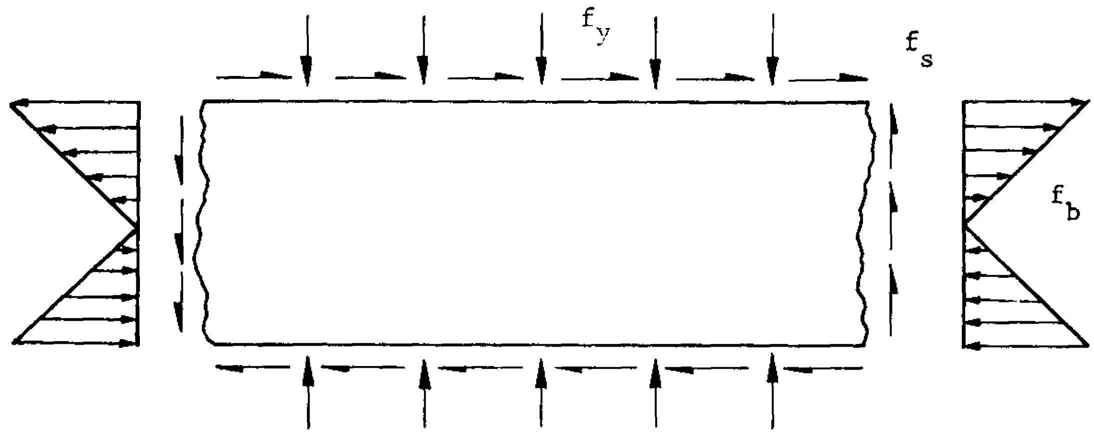
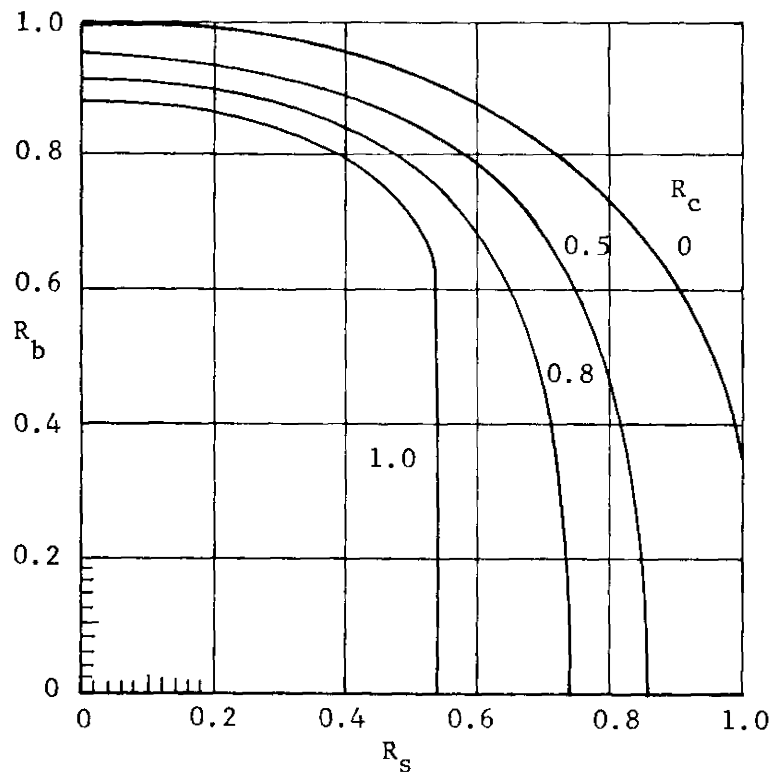


Fig. A 3.5.0-2 (Cont'd)

A 3.5.0 Buckling of Rectangular Flat Plates under Combined Loading (Cont'd)

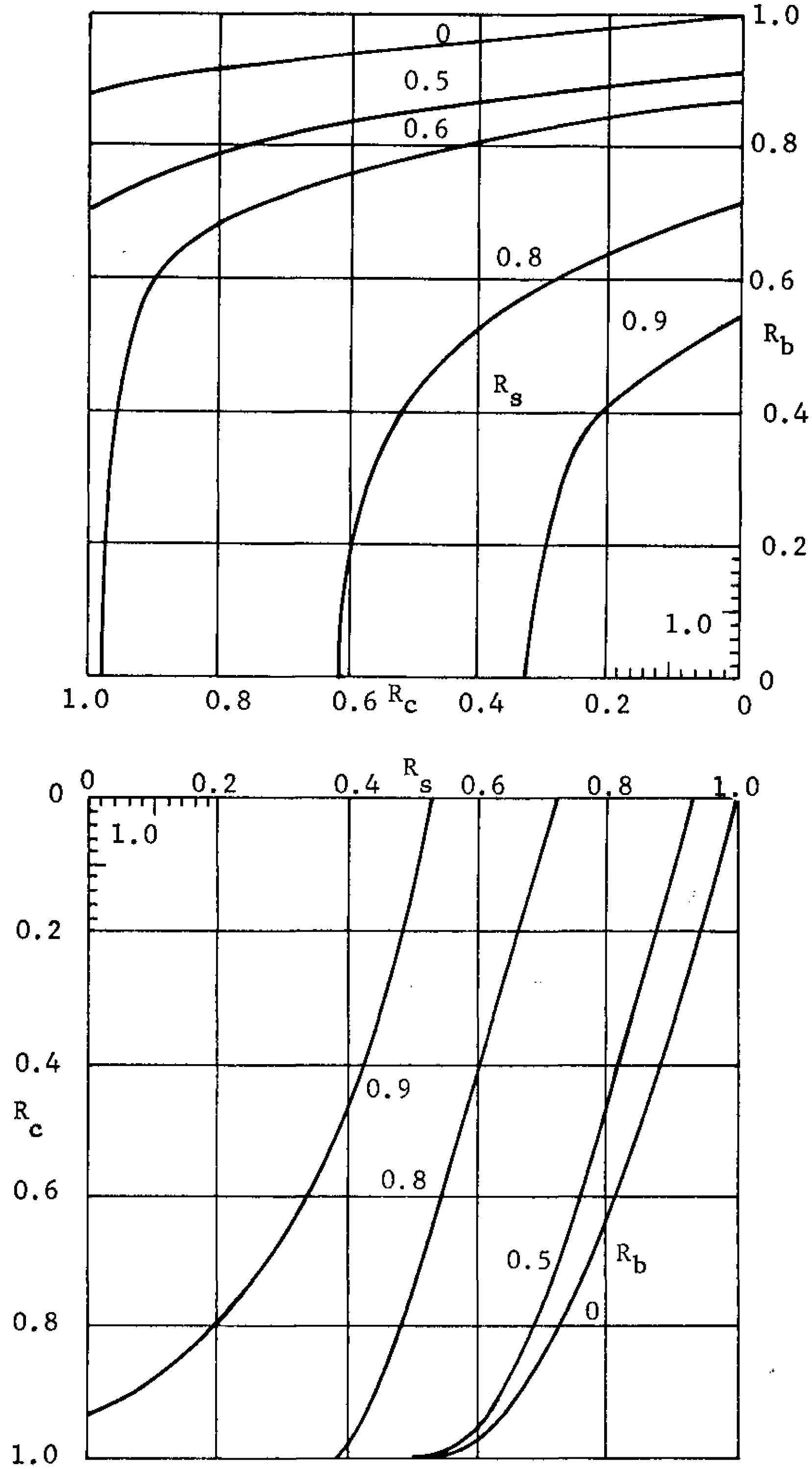




Interaction Curves for Simply Supported Long Flat Plates
Under Various Combinations of Shear, Bending, and Transverse Compression

Fig. A 3.5.0-3

A 3.5.0 Buckling of Rectangular Flat Plates under Combined Loading (Cont'd)



Interaction Curves (Cont'd)

Fig. A 3.5.0-3

Table A 3.6.0-1

STRUCTURE	LOADING COMBINATION	FIGURE	INTERACTION EQUATION	EQ. FOR FACTOR OF SAFETY	NOTES
Curved Plates	Longitudinal Compression and Shear	A 3.4.0-1	$R_s^2 + R_c = 1$	$\frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}}$	
	Longitudinal Compression and Internal Pressure	A 3.6.0-1	$R^2 - \lvert R_p \rvert = 1$		
	Shear and Internal Pressure		where $R = R_c$ or R_s		1
Circular Cylinders	Longitudinal Compression and Pure Bending	A 3.4.0-1	$R_c + R_b = 1$	$\frac{1}{R_c + R_b}$	
	Longitudinal Compression and Torsion	A 3.4.0-1	$R_c + R_{st}^2 = 1$	$\frac{2}{R_c + \sqrt{R_c^2 + 4R_{st}^2}}$	
	Torsion and Longitudinal Tension	A 3.6.0-1	$R_{st}^3 - \lvert R_t \rvert = 1$		2
	Pure Bending and Tension	A 3.4.0-1	$R_b^{1.5} + R_t = 1$		
	Pure Bending and Transverse Shear	A 3.4.0-1	$R_b^3 + R_s^3 = 1$		3

Table A 3.6.0-1 (Cont'd)

STRUCTURE	LOADING COMBINATION	FIGURE	INTERACTION EQUATION	EQ. FOR FACTOR OF SAFETY	NOTES
Circular Cylinders (Cont'd)	Longitudinal Compression, Pure Bending and Transverse Shear		$R_c + \sqrt[3]{R_s^3 + R_b^3} = 1$		3
	Pure Bending Torsion, and Transverse Shear		$R_b^p + (R_s + R_{st})^q = 1$		4
	Longitudinal Compression, Pure Bending Transverse Shear, and Torsion		$R_c + R_{st}^2 + \sqrt[3]{R_s^3 + R_b^3} = 1$		3
	Longitudinal Compression, Pure Bending and Torsion		$R_c + R_b + R_{st}^2 = 1$	$R_c + R_b + \sqrt{(R_c + R_b)^2 + 4R_{st}^2}$	
Elliptical Cylinders	Bending and Transverse Shear	A 3.4.0-1	$R_b^2 + R_s^2 = 1$	$\frac{1}{\sqrt{R_b^2 + R_s^2}}$	3
	Bending and Torsion	A 3.4.0-1	$R_b^2 + R_{st}^2 = 1$	$\frac{1}{\sqrt{R_b^2 + R_{st}^2}}$	

Table A 3.6.0-1 (Cont'd)

NOTES:

1. $R_p = \dfrac{\text{internal pressure}}{\text{external collapsing pressure}}$

2. $R_t = \dfrac{\text{applied tensile stress}}{\text{compression buckling stress}}$; $R_t < 0.8$

3. Use f_s and f_b each as maximum calculated values even though the locations of the two maxima do not coincide. Use F_s as smaller of F_{su} and 1.25 F_{st}. Use buckling stress in bending for F_b.

4. When $\dfrac{R_{st}}{R_s} \leq 1$:

$$p = 3 - \frac{3}{4}\left(\frac{R_{st}}{R_s}\right)$$

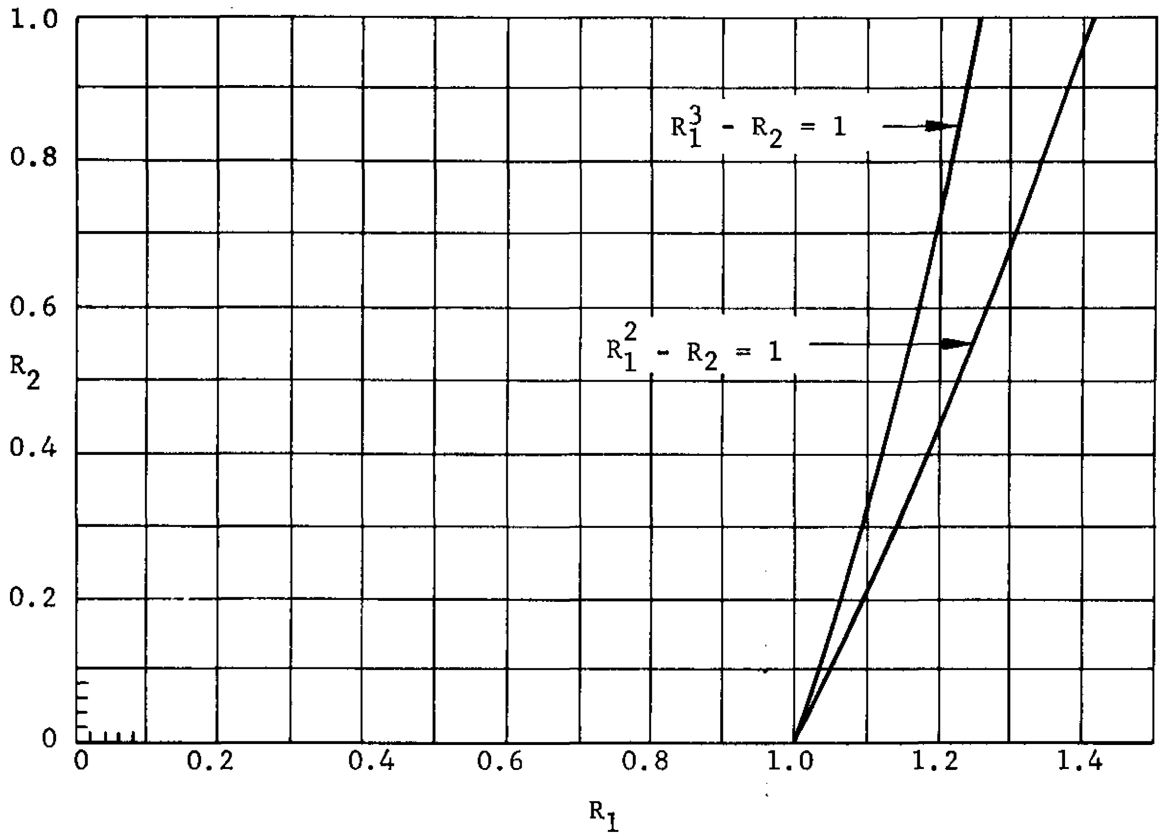
$$q = 3 - \frac{1}{2}\left(\frac{R_{st}}{R_s}\right)$$

When $\dfrac{R_s}{R_{st}} \leq 1$:

$$p = 1.5 + \frac{3}{4}\left(\frac{R_{st}}{R_s}\right)$$

$$q = 2 + \frac{1}{2}\left(\frac{R_{st}}{R_s}\right)$$

A 3.6.0 <u>Buckling of Circular Cylinders, Elliptical Cylinders, and Curved Plates under Combined Loading (Cont'd)</u>



Interaction Curves

Fig. A 3.6.0-1

A3.7.0 Modified Stress-Strain Curves Due to Combined Loading Effects

An analysis that uses a uniaxial stress-strain curve or material properties derived from such a curve (analysis of beams, columns, thermal effects, plastic bending, elastic and plastic buckling, Elastic-Plastic Energy Theory of Section B4.5.7, etc) may require a modified stress-strain curve or properties derived from a modified curve when combined loading is involved. Loads or stresses in one plane affect the loads and stresses in other planes due to the Poisson effect. For example, a tension member fails when the average stress, (P/A), reaches the ultimate tensile stress F_{tu} of the material, but a member resisting combined loading may fail before the maximum principal stress reaches F_{tu} (Reference Section A1). When buckling or other empirical parameters include combined loading effects, modified stress-strain curves are not required.

Several methods of modifying uniaxial stress-strain curves have been developed; the method presented here is derived from the Octahedral Shear Stress Theory.

Assumptions & Conditions:

1. f_1, f_2 and f_3, the three principal stresses, are in proportion; i.e.,

$$f_2 = K_1 f_1 \quad (1)$$

$$f_3 = K_2 f_1 \quad (2)$$

$$K_1 \neq K_2$$

See Fig. A3.7.0-1 for direction of principal stresses.

A3.7.0 Modified Stress-Strain Curves Due to Combined Loading Effect (Cont'd)

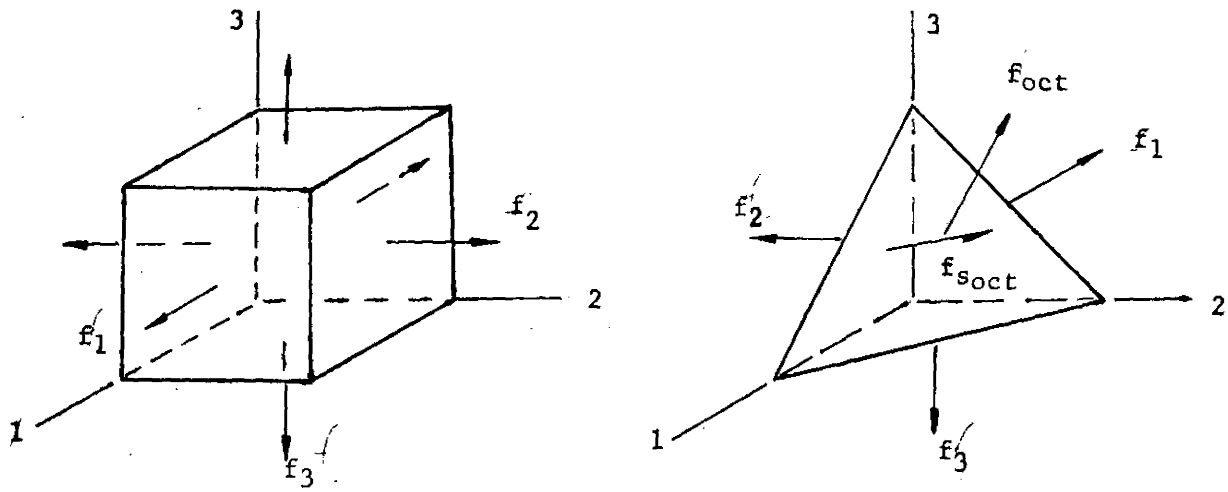


Figure A3.7.0-1

Directions of Principal Stresses

2. Prime (') denotes a modified value:

ϵ' = modified strain

E' = modified modulus of elasticity.

3. In this method, for any principal stress f_i, the total strains and modulus of elasticity are modified to include the effects of the other principal stresses.

Procedure:

1. Calculate the principal stresses for a given load condition (Reference Section A3.1.0).

2. Determine the effective uniaxial stress:

$$\bar{f}_1 = \frac{1}{\sqrt{2}}\sqrt{(f_1 - f_2)^2 - (f_2 - f_3)^2 + (f_3 - f_1)^2} \qquad (3)$$

A3.7.0 Modified Stress-Strain Curves Due to Combined Loading Effect (Cont'd)

and calculate an effective modulus of elasticity, E'_1, by:

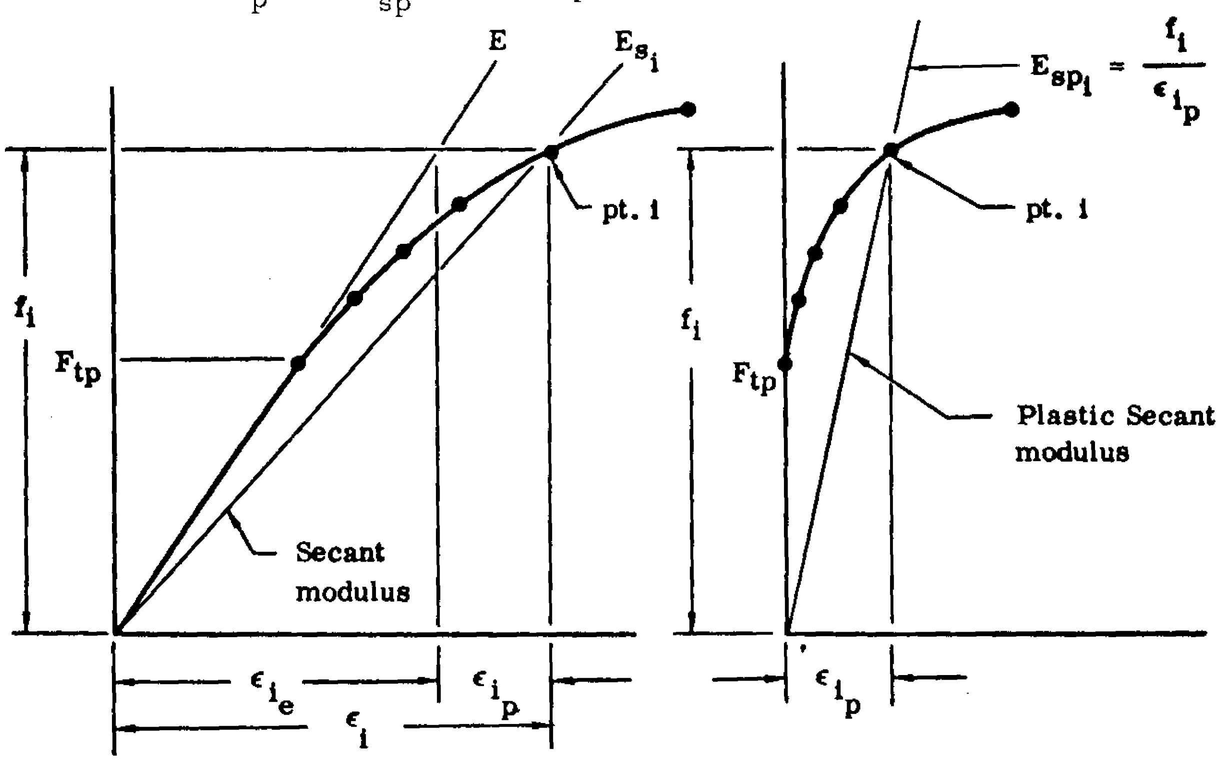
$$E'_1 = \left(\frac{f_1}{\bar{f}_1}\right) E_1 \qquad (4)$$

3. Enter the plastic stress-strain diagram for simple tension of the material, if available, at the value of $\bar{f}_1$ and determine E'_{sp}. (See Figure (A3.7.0-2b) Otherwise, enter the simple tension stress-strain curve at $\bar{f}_1$ and determine E'_{sp} (see Figure A3.7.0-2a) by:

$$E'_{sp} = \frac{\bar{f}_1}{\epsilon_1 - \epsilon_{1_e}} \qquad (5)$$

4. Use this value of E'_{sp} and a value of $\mu_p = 0.5$, if not accurately known, find e'_{1_p} from

$$\epsilon'_{1_p} = \frac{1}{E'_{sp}} (f_1 - \mu_p f_2 - \mu_p f_3) \qquad (6)$$



(a) Engineering stress-strain curve (b) Plastic stress-strain curve

Figure A3.7.0-2

A3.7.0 Modified Stress-Strain Curves Due to Combined Loading Effect (Cont'd)

5. Once E_1' has been found, ϵ_{1_e}' can be determined for any value of f_1 by:

$$\epsilon_{1_e}' = \frac{f_1}{E_1'} \tag{7}$$

6. Determine the total effect strain, ϵ_1', for each value of f_1 by:

$$\epsilon_1' = \epsilon_{1_e}' + \epsilon_{1_p}' \tag{8}$$

7. Repeat all steps until sufficient points are obtained to construct a plot of f_1 vs ϵ_1' (see Figure A3.7.0-3) which is the modified stress-strain curve.

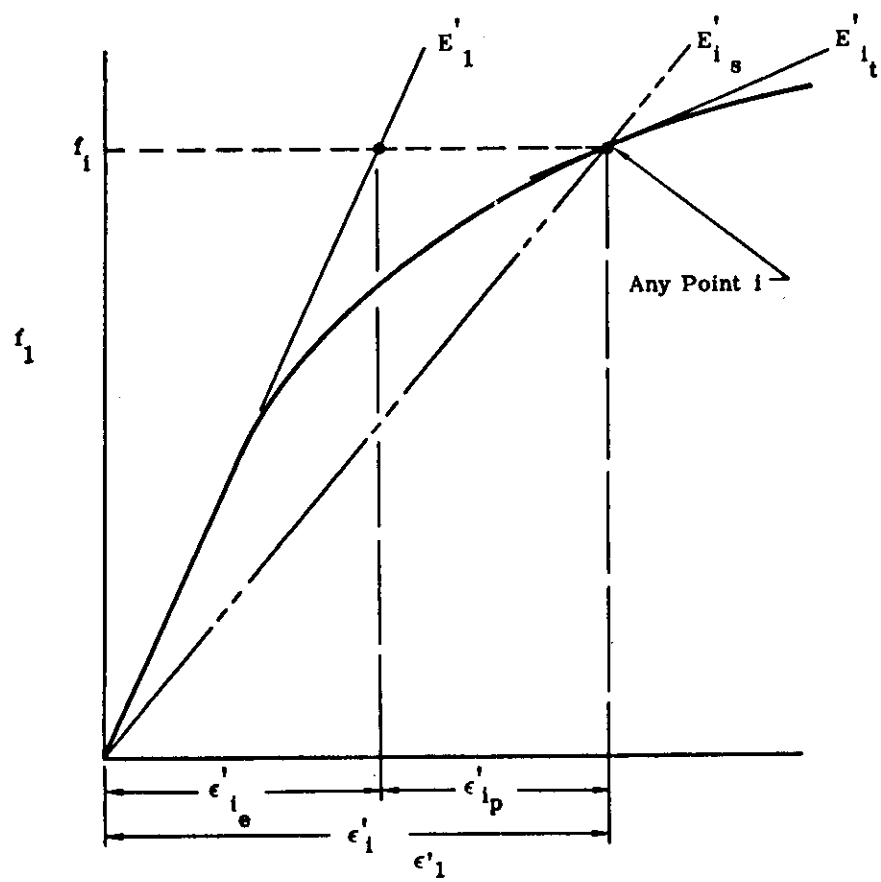


Figure A3.7.0-3 Modified Stress Strain Diagram Due to Combined Loading

References:

Popov, E. P., Mechanics of Materials, Prentice-Hall, Inc., New York, 1954.

Structures Manual, Convair Division of General Dynamics Corporation, Fort Worth.